

This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

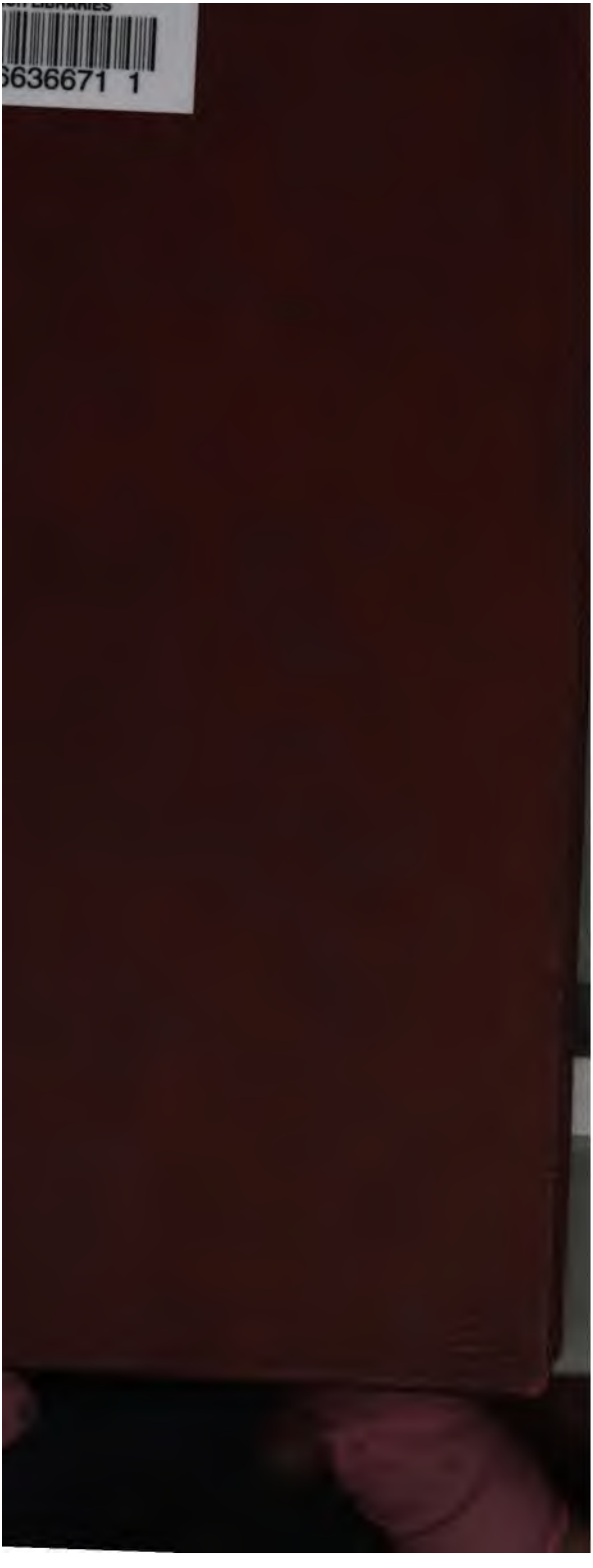
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`


5636671 1





TREATISE

ON

WATER-WORKS.

TREATISE

ON

WATER-WORKS.

A

TREATISE

ON

WATER-WORKS

FOR

CONVEYING AND DISTRIBUTING SUPPLIES OF WATER;

WITH

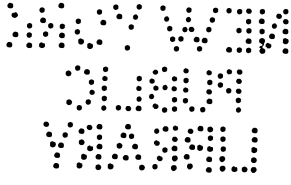
TABLES AND EXAMPLES.

By CHARLES S. STORROW.

NEW YORK PUBLIC
ASTOR LIBRARY
NEW-YORK

BOSTON:
HILLIARD, GRAY, AND CO.
1835.

Entered according to the act of Congress in the year 1835,
by Charles S. Storrow,
in the Clerk's office of the District Court of the District of Massachusetts.



CAMBRIDGE:
CHARLES FOLSOM,
PRINTER TO THE UNIVERSITY.

PREFACE.

In offering a work of this nature, I can of course make few claims to originality; nor has it been my object to produce an original treatise. But I have endeavoured to present the theories of the most eminent philosophers and engineers on the subject of Hydraulics in such a form as to be easily understood, and to show in a simple manner how they should be applied, in solving questions which are, or soon will be, of daily occurrence to the engineers of this country. The works which I have principally followed are, the *Recherches Physico-mathématiques sur la Théorie des Eaux Courantes*, and the *Recueil de Cinq Tables pour faciliter les Calculs relatifs au Mouvement des Eaux*,

by M. de Prony; and the *Essai sur la Solution numérique de quelques Problèmes relatifs au Mouvement permanent des Eaux Courantes*, by M. Bélanger; together with the *Essai sur les Moyens de conduire, d'élever et de distribuer les Eaux*, by M. Génieys. The last work is more appropriate as a practical guide than either of the others, and it owes much of its most valuable matter to M. Bélanger.*

As my object was to render this little treatise practically useful, I have not hesitated, where I have found in these authors such passages as suited my purpose, to translate them, and incorporate them at once into the body of the work. The necessity of reducing the numerical values, so as to make them accord with English measures, has involved a good

* The two publications by M. de Prony can no longer be procured even in Paris; and the two others, having appeared since 1828, have never, so far as my knowledge extends, been noticed in any English work.

deal of labor, and still more so the preparation of the Tables, which it was necessary to calculate entirely anew. I am not aware of the existence of these Tables in any English or American publication; at least I have often sought for them in vain, when wanting them for my own use.

The Chapter on Artesian Wells is taken from Garnier's Treatise on this subject, with the exception of the parts which relate to this country. I have also consulted the articles in Rees's and Brewster's Encyclopædias, the Memoirs of the Academy of Sciences of Paris, the Philosophical Transactions of the Royal Society, the Memoirs of the American Academy of Arts and Sciences, the Franklin Journal, Silliman's Journal, and a number of Reports by various Engineers.

C. S. S.

Boston, March 1, 1835.

Errata.

Page	41,	*line* 16,	*after* v	*read*	$=$
	99,	" 4,	*for* parts	"	parts,
	120,	" 11,	" 0.000242651	"	0.0000242651
	124,	" 6,	" no	"	an
	128,	*col.* 5, *line* 8,	*for* 301.8	"	310.8
	197,	*line* 15,	*for* B'	"	B
	199,	" 18,	" HD to ID	"	ID to HD
	202,	" 21,	" d^5	"	d'^5
	224,	" 39,	" 11.30	"	0.30

CONTENTS.

INTRODUCTION.

PAGE

THEORIES OF SOME OF THE EARLIER WRITERS ON THE SUBJECT OF FLOWING WATERS . . . 1

CHAPTER I.

THEORY OF THE MOTION OF WATER IN OPEN CHANNELS.

Properties of fluids. — Retarding forces. — Investigation of a formula to represent the uniform motion of water. Investigation of a formula for the permanent motion of water. — Determination of the constant coefficients. — Recapitulation. — Table 17

CHAPTER II.

THEORY OF THE MOTION OF WATER IN PIPES.

Investigation of a formula to represent the motion of water flowing through pipes. — Determination of the constant coefficients. — Simpler formula. — Error to which it may lead. — Simpler formula for finding the discharge of pipes. — Amount of error to which it may lead. — Eytelwein's formulæ. — Recapitulation. — Tables. — Effect of sudden contractions or

PAGE
enlargements. — Of bends. — Of the air. — Girard's experiments on the motion of water in very small tubes 48

CHAPTER III.

General Remarks on the Means of Supplying Cities with Water.

Quantity of rain in various parts of the United States. — Quantity of water furnished to a certain population in various localities. — Quality of water. — Comparative advantages of various modes of conveyance 71

CHAPTER IV.

Means of Measuring the Flow of Water.

Formula for the efflux from a vessel. — Contraction of the fluid vein. — Table of heights due to various velocities. — Various methods for measuring the discharge of a stream. — Relation between the mean velocity and that at the surface. — Table of mean and surface velocities. — Means of measuring the velocity at the surface. — Quantity flowing over dams. — Scale of velocities 81

CHAPTER V.

Conveyance of Water by Canals or Aqueducts.

Mean radius when the section is trapezoidal. — Leakage. — Proper velocity. — Influence of plants and herbage on the velocity. — Aqueducts of masonry. — Examples. — Remarks on

PAGE

the formula of Article 24. — Motion of water in a horizontal channel. — Recapitulation. — Case in which there are several successive derivations from the main stream. — Recapitulation 104

CHAPTER VI.

CONVEYANCE OF WATER BY CONDUIT PIPES.

Examples. — Flow by a single pipe. — To find velocity, discharge, or diameter. — Quantity delivered by the same pipe at various heights. — Influence of a bend. — On drawing water from two different reservoirs. — Example. — Obstructions by the air, and the remedies. — Materials for making pipes. — Pressure. — Thickness of pipes. — Comparative advantages of one or two pipes. — Joints. — Contraction and expansion. — Means of replacing defective pipes. — Caulking materials and cement. — Mode of laying pipes. — Discharging cocks and stop-cocks. — Table of the weight of cast iron pipes 144

CHAPTER VII.

OF PUMPS.

Lifting pump. — Forcing pump. — Performance of steam-engines 175

CHAPTER VIII.

RESERVOIRS, AND PIPES FOR DISTRIBUTING WATER.

Filtering beds. — Investigation of formulæ for the motion of water in a system of main and

PAGE

branch pipes. — Example to show their application 191

CHAPTER IX.

PIPES OF DIFFERENT DIAMETERS, AND JETS D'EAU.

Formula for the motion of water through pipes varying in diameter. — Example. — Formula for the motion of water in vertical jets. — Example 213

CHAPTER X.

ARTESIAN WELLS.

Sections of the strata bored through in various localities. — Formations in which we should seek for rising springs. — Wells in Boston . 222

APPENDIX. 239

INTRODUCTION.

An abundant supply of pure and wholesome water is every where one of the first requisites for health, cleanliness, and comfort. It is especially so in crowded cities, where, in addition to the quantity required for domestic purposes, it is highly important to possess a sufficient supply to check the progress of conflagration, to cleanse the streets from the accumulation of offensive matters, and to purify and cool the air by a copious distribution from fountains and *jets d'eau.*

The art of supplying the deficiency of nature by conveying this precious element in plentiful streams from a distant source, does not owe its origin to the luxury of modern times, as is the case with so many of the arts, which now minister to our physical enjoyment. On comparing the hydraulic works of the ancient Romans with those erected within a comparatively recent period, we find them still unsurpassed in grandeur and magnificence. At a wonderful expense of labor, they carried their aqueducts through hills and over valleys, in spite of every natural obstacle;

and it will give some idea of the scale on which these noble monuments were constructed, to mention, that, according to the *Commentary* of Frontinus, who, in the year 98, had the direction of the nine aqueducts by which water was conveyed to Rome, their united length amounted to 250 miles, and their daily supply to more than 27,000,000 cubic feet. This author adds, that the supply might have been doubled by proper care to prevent fraudulent usage.

Nor was it for the Imperial city alone, that the Romans sought to procure this blessing. In the various provinces of their empire they constructed works of the same kind on the same magnificent scale. Similar achievements are not to be expected at the present day, when happily it is no longer the prerogative of a single nation to wield the strength and apply the resources of the whole civilized world. But it is the boast of modern science, that it supplies the want of immense physical forces by the most ingenious application of the means which it can command; and, instead of overcoming obstacles, as was once done, by dint of the combined and persevering labor of thousands, it skilfully eludes the difficulties which lie in its path, and finds its glory no less in the feebleness of the means by which it accomplishes great objects, than in the importance of the objects themselves.

It seems hardly necessary to enlarge upon the advantage to be derived from a careful consultation of the most improved theory, when there is any question of constructing works of the kind which we have been describing. At the present day, when we see so many ways of applying the wealth and resources of civilized communities to the amelioration of their condition, and the partisans of each are loudly proclaiming its superior advantages, it is of the greatest importance not to waste in rash and hasty enterprises those means, which, employed in some other manner, might have been productive of happy effects. When there is no experiment to furnish a guide, the object of theory is, to direct us to those measures, which have the highest probability of success to recommend them; and when we have the benefit of previous experiments, it teaches us the proper manner of employing them, by pointing out the anomalies, and showing us how to distinguish the influence of the various causes, which act in combination.

That part of the science of Hydraulics, of which the construction of water-works is an immediate practical application, has, from its importance, engaged the attention of many eminent men; and there are few more beautiful examples of the application of mathematical analysis to the discovery of physical truths, than are to be found in

the writings of some of them. Though very far from having reached that perfection to which the labors of subsequent experimenters will undoubtedly carry the science, yet the results already obtained are so satisfactory, that it would be folly to neglect them, before the united efforts of theory and practice have furnished us with a safer guide. It is true, that many experiments are yet to be desired in order to confirm some of the results which theory alone has yet announced. The formulæ recently given by Bélanger should be subjected to this test, and also the rules given for calculating the motion of water which flows through a system of branch pipes. The engineers having charge of the various hydraulic works, which will doubtless ere long be commenced in this country, as the principal cities increase in wealth and population, and feel more and more the necessity of procuring ample supplies of water, will find many opportunities of adding to the experimental results which we already possess; and it should be remembered that even a single experiment, carefully and judiciously made and faithfully reported, is a valuable present to science. We see this strikingly, when we read some of the early works on hydraulics. The *theories* of the authors have in many instances been entirely exploded, or greatly modified and improved, by later inquirers; but their

experiments can never grow old. Until the laws of nature lose their permanency, what was a fact in one age is equally a fact in another, and no succeeding observations can set it aside.

The object of the present treatise is, to exhibit known results in as simple a manner as possible, and to show their application as required by the Engineer charged with the execution of hydraulic works. It seems proper to begin by briefly recounting the various steps, by which this branch of science has advanced to its present state, and we shall do this in a small compass. We shall afterwards give an exposition of the theory of the motion of flowing waters, taken from the works of Prony, Eytelwein, and Bélanger, by whom it has been chiefly perfected, and then proceed to the practical application of the principles thus established.

In Italy, where at a very early period all the resources of art were called for to restrain within their allotted bounds the numerous torrents, which rush down from the mountains, and to protect from their ravages the fields and dwellings of the inhabitants, the study of hydraulics was eagerly pursued by a host of diligent and talented men. But notwithstanding their industry, and the large number of works which exist to bear witness to the extent of their labors, the subject was but little understood until within a very recent date.

In the year 1684, was published the *Traité du Mouvement des Eaux*, by Mariotte. This author, following a course better adapted than that of his predecessors to lead to useful practical results, sought to put his theory to the test of experiment. He made numerous practical observations on the motion of water when flowing in tubes, and from orifices of various kinds, and he gave rules for determining the height to which a vertical jet will rise in the air, when connected with a reservoir maintained at a constant height. The experiments made by him have since been repeated with greater care, but we must not refuse him the credit of having pointed out the true mode of investigation.

In the year 1732, Couplet made, and communicated to the Academy of Sciences of Paris, a number of experiments on the conduit-pipes, which conveyed water to the gardens of Versailles. Though seemingly intended only to minister to the luxury of Louis XIV., these magnificent works had an important effect upon the advancement of this branch of science, because they called the public attention more strongly to the subject, and furnished at the same time an opportunity of making experiments on a very large scale. Couplet showed the astonishing discrepancy between the practical results obtained, and those which would have been indicated by

theory as it then was, and thus proved the necessity of a great modification of this theory. He investigated the resistance offered by the air which collects in pipes, wherever there are vertical flexures; and even at the present day his experiments, which may be found at length in the Memoirs of the Academy of Sciences of Paris, for 1732, are of great value, however little may be the importance of his theoretical views.

Soon after this, the theory of hydraulics was established by Daniel Bernouilli and by D'Alembert upon principles far more accurate than any by which their predecessors had been guided. Adopting hypotheses, which appear very conformable to nature, they were able to express algebraically the conditions of the motion and equilibrium of fluids, and to render the solution of many questions dependent only upon the progress of purely mathematical science.

In the year 1771, the Abbé Bossut published his *Theoretical and Experimental Treatise on Hydrodynamics*. This work, the most valuable of those which had yet appeared, contains a very large number of experiments made with the greatest care, and adapted to furnish a guide in the cases which are of the most common application. Those which relate more particularly to the subject before us, were made with a view to determine, with greater accuracy than had yet

been done, the value of the coefficient of contraction of the fluid vein, as water flows from an orifice in a thin plate, or in additional tubes of various lengths and figures, the motion of water in conduit-pipes of considerable length for various diameters and inclinations, and the motion in open canals and rivers. The experiments of Bossut were on a small scale, it is true; yet their great accuracy, and the judgment which he manifested in his selection of the questions to be submitted to the test of experiment, place him among the highest on the list of those to whom the science of hydraulics owes its present advancement.

It being now ascertained experimentally, that the motion of water in conduit-pipes or in open channels is influenced by retarding forces, whose effect is very great, Chézy, a French engineer, who in 1775 was studying a project for bringing the waters of the Yvette into Paris, undertook to express in an algebraical formula the laws of the motion of running water, taking into account these retarding forces. According to him, the retarding forces are proportional to the length of the canal, to the perimeter of the section in contact with the water, and to the square of the velocity. He came at length to the very simple equation, $v = \sqrt{\frac{g\,\omega\,\zeta}{\beta\,\lambda\,\chi}}$, in which v represents the mean velocity, g the accelerating force of gravi-

ty, ω the area of the section of the stream, ζ the difference of level between its two extremities, λ its length, χ the perimeter of the portion of the channel in contact with the water, β a constant coefficient to be determined by experiment.

In 1786, Dubuat published his *Principes d'Hydraulique;* this work was the result of ten years' labor. Following the example of Bossut, and supporting by experiment every thing which he advances, Dubuat has furnished a most important guide to practical engineers. He first proposes an hypothesis, suggested by analogy or by a careful consideration of the various causes which may influence the motion of a stream of water, and then, calling experiment to his aid, he successively modifies his formulæ, until he has brought them to represent, with sufficient accuracy, the actual results. His first object was to solve the problem, which he calls the key of hydraulics; namely, to determine what will be the velocity of a stream, when the section of the channel and its declivity are known. After establishing as principles in the course of his investigation; 1. that the moving force of each of the particles of water which compose a river arises simply from the inclination at the surface; and 2. that, when water is flowing in a uniform manner, the resistance which it experiences is equal to its accelerating force; he arrives at the formula

$$V = \frac{307\,(\sqrt{r} - 0.1)}{\sqrt{b} - \text{Log.}\sqrt{b} + 1.6} - 0.3\,(\sqrt{r} - 0.1),$$

in which the letters represent the following quantities, the English inch being the unit.

V.... the mean velocity per second, in inches, of a stream of water flowing in a channel of indefinite length, the section and the inclination being constant.

r.... the mean radius, by which is meant the quotient of the section of the stream, divided by that part of the perimeter of the channel, which is in contact with the water. In a circular pipe, the mean radius would therefore be one fourth of the diameter.

b.... the denominator of the fraction expressing the slope of the channel, the numerator being 1. Thus a slope of 1 inch in 1,000 would be represented by $\frac{1}{1000}$ and $b = 1{,}000$.

Log. designates the hyperbolic logarithm of the quantity to which it is prefixed, and can be obtained by multiplying the common logarithm by 2.30258.

This formula, as may be seen by the experiments of Dubuat, represents the effective results with a very great degree of accuracy; but it is of a somewhat complicated form, and contains a logarithmic quantity which renders it rather in-

convenient in its application.* Dubuat next proceeds to apply this useful formula in various ways, and to show of what vast importance it is in determining questions relating to the flow of rivers, the effect of alterations which may be made in their channel, the influence of sinuosities and of various obstructions to the regularity of their course; and afterwards enters upon an investigation of the motion of water in conduit-pipes and in *jets d'eau*, always confirming his views by showing their coincidence with numerous experiments.

Smeaton, whose accuracy and care impart a value to all his experiments, turned his attention to this subject. Taking a pipe 100 feet long and $1\frac{1}{4}$ inches in diameter, he ascertained experimentally the depth of the column necessary to impel the water through it at various velocities from 0.50 feet to 3.50 per second, and then assumed that the depth of the column in pipes of other dimensions was as the length of the pipe directly, and as the diameter inversely. From these experiments the following table was constructed.

* A number of tables have been computed by Mr. John Lourie of Glasgow, to facilitate the application of Dubuat's formula, and may be found in the Edinburgh Encyclopædia, *art.* Hydrodynamics.

Velocity in feet per second.	Length 100 ft. Bore 1 inch. Depth required in inches.	Velocity in feet per second.	Length 100 ft. Bore 1 inch. Depth required in inches.	Velocity in feet per second.	Length 100 ft. Bore 1 inch. Depth required in inches.
0.083	0.12	1.250	12.5	2.666	48.2
0.166	0.40	1.333	14 2	2.833	54.2
0.250	0.6	1.416	15.7	3.000	60.5
0.333	1.0	1.500	17.5	3.166	67.2
0.416	1.6	1.583	19.2	3.333	74.5
0.500	2.2	1.666	21 0	3.500	82.0
0.583	3.0	1.750	22.9	3.666	90.0
0.666	4.0	1.833	24.7	3 833	98.2
0.750	4.7	1.916	26.9	4.000	107.0
0.833	5.9	2.000	28.9	4.250	120.6
0.916	7.1	2.166	33.2	4.500	135.4
1.000	8.4	2.333	37.9	4.750	151.7
1.083	9.7	2.500	42.9	5.000	168.0
1.166	11.1				

The above table gives the velocities and head, when the length is 100 feet, and the bore 1 inch. If the length be 200 or 300 feet, the head must be double or triple. If the bore be $\frac{1}{2}$ inch the head must be double; if the bore be 2 inches, the head must be one half of that given in the table. The experiments were not sufficiently varied, nor numerous enough for us to use the table with that confidence, which the name of Smeaton usually inspires.

Such being the state of the science at the close of the last century, in order to find a correct formula for the motion of water, it seemed to be an important object to ascertain more precisely the nature and the influence of the retarding forces, and to find for them an analytical expression both simple and accurate. In 1800,

Coulomb proved * that if the retarding forces were expressed by a function composed of two terms only, one of which is proportional to the first, and the other to the second power of the velocity, the motion deduced from theory would agree with that of experiment at moderate velocities. This was a very important discovery, and has powerfully contributed to advance the science. Mr. Girard a French engineer, who had charge of the execution of the Canal de l'Ourcq, which brings water to Paris, was the first person who applied Coulomb's researches in finding a formula for the motion of flowing water. He expresses the resistance due to cohesion by the term $R' \chi v$, and that due to the resistance occasioned by the roughness of the channel by $R' \chi v^2$; hence adding the two expressions he obtains $R' \chi (v + v^2)$ for the sum of the resistances, and

$$\frac{g \omega \zeta}{\lambda \chi} = R' (v + v^2)$$

for the formula of the motion, R' being a constant coefficient to be determined by experiment, and the other letters representing the same things as in the formula of Chézy given above. From a number of experiments Girard fixed the value of R' at 0.0012181, the unit being the metre.

* Mémoires de l'Institut.

In 1804, M. de Prony published his *Physico-Mathematical Researches on the Theory of Flowing Waters*, in which, after a profound theoretical investigation of the proper form of the function expressing the retarding forces, he finds it necessary to give a different coefficient to the two terms, to both of which, Girard had given the same coefficient R'. Selecting a large number of the most accurate experiments of Couplet, Bossut, and Dubuat, he then determines by means of them the numerical value of these coefficients. This formula unites to its simplicity and facility of application, the advantage of representing more accurately the results of experiment, than those of any of his predecessors.

In the Philosophical Transactions of the Royal Society, for the year 1808, was published a paper by Dr. Thomas Young, in which he proposes the following formula to represent the motion of flowing water;

$$v = \sqrt{-\left(\frac{d}{a}s + \frac{c^2}{a^2}\right)} - \frac{c}{a}.$$

In which v represents the mean velocity per second;

d 4 times the mean radius,

s sine of inclination, or the head divided by the length;

$a, . . c, . .$ quantities, which must be

determined for various values of d. The unit being the English inch.

He makes

$$a = 0.0000001 \left(430 + \frac{75}{d} - \frac{1440}{d+12} - \frac{180}{d+\frac{1}{3}}\right)$$

$$c = 0.0000001 \left(\frac{900\,d^2}{d^2+1000} + \frac{1}{\sqrt{d}}\left(1050 + \frac{12}{d} + \frac{0{,}9}{d^2}\right)\right).$$

In the same paper he has given the values of a and c already calculated for 44 different values of d, from $d = 0.1$ to $d = 500$. The greater part of them are contained in the following table.

d	a = 0,0000001 ×	c = 0,0000001 ×	d	a = 0,0000001 ×	c = 0,0000001 ×
0	413	900	15	354	430
500	410	944	10	339	413
400	409	948	9	336	421
300	406	951	8	331	433
200	404	951	7	327	449
100	399	918	6	322	471
90	398	903	5	312	507
80	396	885	4	306	556
70	393	860	3	292	635
60	391	825	2.5	284	694
50	389	772	2	277	774
40	383	698	1.5	266	894
30	377	597	1	251	1099
25	371	526	0.5	245	1578
20	364	482			

Eytelwein, a German hydraulician, published the results of his experiments in 1814 and 1815,

* When $d = 500$ we have

$a = 0.0000001 \times 410 = 0.0000410.$

$c = 0.0000001 \times 944 = 0.0000944.$

in the Memoirs of the Academy of Berlin. Following in the steps of Prony, but having a greater number of experiments to guide him, he has given new values to the coefficients first determined by his predecessor. The difference, however, is so small as to have but very little influence upon the results within the common limits of application.

An account of the labors of Prony and Eytelwein, together with a valuable paper by Bélanger, published in 1828, will be given at length in the following chapter.

CHAPTER I.

THEORY OF THE MOTION OF WATER IN OPEN CHANNELS.

1. It is a fundamental property of fluids to transmit pressure equally in all directions. If a pressure be applied by means of a piston upon the surface of the water contained in a vessel of any shape whatever, any portion of the sides or bottom of the vessel, equal in extent to the area of the piston, supports (in addition to the effort due to the weight of the water) exactly the same pressure as if the piston were applied directly to it.

The pressure which the water exerts in virtue of its weight increases with the depth below the surface, and is equal, upon any portion of the sides or bottom, to the weight of a column of water having for base the area of this portion, and for height the vertical distance of its centre of gravity from the surface.

The atmosphere exerts a pressure also upon all water to which it has access; so that, if a tube

be placed vertically with one end in the water, and the air withdrawn from it by means of a piston or otherwise, the atmospheric pressure on the external water will be transmitted to that under the tube, which, meeting with no corresponding pressure above it, will rise till it has attained the height of about 34 feet, at which height its weight will counterbalance that of the atmosphere.

2. The particles of which water is composed adhere so feebly to each other, that the slightest difference in the pressures exerted upon them is sufficient to set them in motion, and they will always flow in that direction where the pressure is the least. It is in consequence of this that water flows in the channels of rivers, through canals when it has a slope at the surface, and through pipes; yielding in the two first to the force of gravity, and in the latter case sometimes to the force of gravity, and sometimes to an impulse given by mechanical means. Indeed this extreme mobility causes it to flow, not only on a descending, but even on a horizontal or rising plane, when the *surface* descends in the direction of the motion. In this case, however, the motion is still the effect of gravity. Each section of the stream being higher than that adjoining to it on the down-stream side, exerts a pressure due to its excess of height, and this is transmit-

ted horizontally in virtue of the known property of fluids. Thus each section is pressed forward by that which is above it on the one side, and presses forward that which is below it on the other, so that the stream flows onward in virtue of the declivity at the surface, whatever may be the inclination at the bottom.

3. When water flows in virtue of its gravity, we might naturally expect its velocity to be uniformly accelerated; and if it possessed neither viscosity nor adhesion, and if the channel were perfectly polished so that there could be no friction, this would certainly be the case. But the fact is far otherwise. Experience shows us that when the supply is constant, and the mode of discharge remains the same, streams of water soon attain a permanent state, and will continue to flow with the same velocity and the same declivity at the surface until some foreign cause begins to interfere. Hence we infer that there are retarding forces, in virtue of which flowing waters presently acquire a uniform instead of an accelerated motion; and this motion may be considered as due, not to gravity alone, but to the combined influence of the force of gravity and these retarding forces.

4. M. de Prony in his *Recherches Physico-Mathématiques sur la Théorie des Eaux Courantes*, published in 1804, from an immediate con-

sideration of the accelerating and retarding forces obtained the formula $RI = av + bv^2$; in which R represents the *mean radius*, as it is called by Dubuat, or the area of the section of the stream divided by the length of the perimeter of the channel in contact with the water; I the sine of the angle contained between the axis of the stream and a horizontal line; v the mean velocity, or the quantity, which, multiplied by the section, gives the discharge; and a and b two constant quantities. In a paper published in 1825,* the same author shows in the following manner the train of reasoning by which we may be led to this equation.

5. When water is flowing in a canal or pipe, a thin lining of fluid particles adheres to the interior surface of the pipe or channel, and does not partake of the general motion. This stationary envelope opposes to the motion of the particles immediately in contact with it, a resistance arising both from adhesion, and from a sort of friction different from that which is observed at the contact of solid bodies, and which, by means of the adhesion, is transmitted from the outside to the interior of the stream. A uniform motion is obtained when, the form of the channel remaining

* Recueil de cinq Tables pour faciliter et abréger les Calculs des Formules relatives au Mouvement des Eaux, &c., par M. de Prony.

invariable, the same quantity of water passes through each section in the unit of time; gravity and the retarding forces thus combining to produce a constant discharge. In such a case, let us now consider the stream as composed of fluid lines parallel to the axis, and both reasoning and observation agree perfectly in the following conclusion; that, the volume of water which flows through any transverse section of the canal or pipe remaining constant, the velocity in each linear element of the stream is the same throughout its whole length, but varies as we pass from one line to another. The velocities then in different points of the same transverse sections are not the same. Thus there is one line in which there will be the greatest velocity, and as we pass to other lines we shall find in them continually decreasing velocities, as they are farther removed from the first. In a cylindrical tube held in a vertical position, it is very evident that the line of greatest velocity will be the axis, and that the velocity will decrease as we approach the circumference. The position of this *maximum* line, however, is not necessarily in the centre of the figure of the section. In open canals, for instance, we should be led to suppose it at the middle of the surface, and experiment shows that this is very nearly its position.

6. Among the various velocities of the different lines which compose the stream, there is one which we call the *mean velocity*. It is such that, if all the lines were endued with it, the volume of water flowing through each section in the unit of time would be exactly the same as that which actually flows now; or, in other words, the mean velocity per second is equal to the volume per second divided by the section.

This is a principal element to be determined in all calculations relating to flowing waters, and we must give an idea of the relations which exist between it and the other elements.

7. It is evident, in the first place, that the mean velocity must increase with the inclination, which last quantity has for its measure the sine of the angle contained between a horizontal line and the axis of the stream; for the greater the inclination the greater is the component of gravity in the direction of the motion. On the other hand the resistance, which acts at the perimeter of the section, must augment with its length. But we know that, without changing this length of perimeter, we may vary the extent of the inscribed surface; and it has been found that, with the same declivity and length of perimeter, the greatest inscribed area corresponds to the greatest mean velocity. This fact might have been presumed *à priori;* for we

might naturally suppose that the influence of the resistance developed at the contact of the stream with the perimeter of the section, supposing this to remain the same, becomes on the whole less sensible in proportion as the volume of water on which it acts is more considerable.

We have then two quantities which increase or decrease at the same time with the mean velocity, namely, the inclination, or the inclination multiplied by the accelerating force of gravity, and the area of the tranverse section; and a third quantity, the length of the perimeter, which varies in a contrary manner. If we denote by I, ω, χ, respectively the sine of the angle of inclination, the area of the section, and the length of the perimeter, g being the accelerating force of gravity, represented by the velocity which a heavy body acquires in a second when falling vertically in a vacuum ($g =$ 32.182 feet), the value of $\frac{g\,\omega\,I}{\chi}$ will increase and decrease with the mean velocity v; and this is a conclusion confirmed by all known experiments, and may therefore be considered as a fact established beyond dispute. It follows from this, that the mean velocity v is a function of $\frac{g\,\omega\,I}{\chi}$ and reciprocally, so that $\frac{g\,\omega\,I}{\chi} = F(v)$, $F(v)$ denoting a function of v, or a quantity which

increases and decreases with v. To determine the form of this function is a capital point. But it results from Coulomb's researches and experiments, that when the motion has become uniform, this function contains only the first and second powers of the velocity; hence it must be of the form $A\,v + B\,v^2$, and we have $\frac{g\,\omega\,I}{\chi} = A\,v + B\,v^2$, A and B being coefficients to be determined by experiment. If we make $\frac{A}{g} = a$, $\frac{B}{g} = b$, g being sensibly constant, and $\frac{\omega}{\chi} = R$, we shall have $R\,I = a\,v + b\,v^2$.

For the determination of the constants a and b, see Art. 25.

8. This formula of M. de Prony can be applied only in those cases where the motion of water is what he calls *uniform;* that is, when the stream may be considered as composed of straight and parallel lines, the elements of which have a velocity differing perhaps in the different lines, but constant in the same line. M. Bélanger, a French engineer, has since given a more general solution of the question, and furnished a formula for the *permanent* motion of flowing waters. This *permanent* motion he thus defines.

* Imagine a canal of a certain length whose sides and bottom are incapable of being moved, or any wise altered, by the stream which flows through them. Let the inclination and the transverse section vary according to any law, provided however that there be no sudden changes of direction which might occasion eddies and undulations in the stream. Suppose, moreover, that the canal is supplied at one extremity by a source which furnishes a constant quantity per second, and that it discharges its waters at the other in a uniform manner, as for instance into a basin whose level is constant, or over a dam, or that it falls freely from the extremity like a cascade. After a certain interval of time from the moment the water is first let into the canal, a current will be established throughout its length, through every section of which there will pass per second exactly the same quantity of water that the source supplies. From that moment the surface of the stream retains invariably the same position, and a section taken at any point is always the same. The stream is now in a state of *permanent motion*; and it is essential not to confound this with the *uniform*

* Essai sur la Solution numérique de quelques Problèmes relatives au Mouvement permanent des Eaux Courantes, par M. J. B. Bélanger.

motion, which is only a particular case of the former.

9. The sole condition of *permanent motion* then is, that the stream may be considered as composed of fluid lines, invariable in form and position, discharging a constant quantity of water in the unit of time, but whose section, and consequently whose velocity, may vary in different points of the same line. It is this last circumstance which distinguishes it from the *uniform motion*, in which the section and velocity remain constant through the whole length of each line.

10. Bélanger now proceeds to find analytically a formula expressing, in the case of *permanent motion*, the relation between the length, the velocity, the section, and the inclination, and begins like Prony by adopting three hypotheses which must be carefully kept in mind.

(1.) Supposing the space occupied by a stream in permanent motion to be divided into infinitely thin sections by a series of planes perpendicular to the axis, we admit that the velocity of all the particles of water, as they pass through any one section, is the same, and that this velocity only varies as we pass from one section to another.

(2.) We suppose each particle to move sensibly in a straight line, so that we may neglect the centrifugal force resulting from a curvilinear mo-

tion, if such there be, as being of no appreciable consequence compared with the other forces which act upon the system.

(3.) The following analysis applies only to those cases where the dimensions of the transverse section of the stream vary by quantities exceedingly small compared with the length, so that we may at each instant regard the velocity of each particle as perpendicular to the section through which it passes, neglecting the velocities in a transverse direction, which do in fact exist, as soon as we admit a variation in the successive sections.

11. Of these three hypotheses, the first, which supposes a motion by whole sections at a time, is never realized in nature; because the effect of the resistance which the sides of the channel oppose to the motion of the water is continually diminished in its transmission from the sides to the centre line of the stream, which will therefore have the greatest velocity (Art. 5.). But as in general the velocities in the different parts of the same section do not differ very much from each other, it is evident that we commit but a slight error in substituting for them the mean velocity, which is the quotient of the quantity discharged in the unit of time, divided by the area of the section in question.

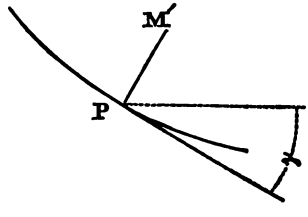
12. This compensation being allowed, each linear element of the stream may be considered

as retarded by a force which acts like friction in a contrary direction to the motion; and M. de Prony, in his *Physico-Mathematical Researches on the Theory of Flowing Waters*, has shown that this force has for its expression the function

$$g \frac{\chi}{\omega} (a v + b v^2),$$

in which the letters denote the same things as in Art. 7.

13. This being premised, let us consider a point M of the stream, vertically projected in M', as in the annexed figure. Through this point suppose a plane normal to the axis and intersecting it in P. This plane is perpendicular to that of the figure, and meets it in the line $M'P$. Retaining the preceding signs, let also



s represent the length of the part of the axis included between the point P and some fixed point on the axis taken higher up towards the source.

y the distance $M'P$ between the particle in question and the horizontal line drawn through the axis in the plane of the transverse section, the projection of this horizontal line being P.

γ the angle which the direction of the axis at P, or what is the same thing, which the direction of the velocity of the stream at M, makes with the horizon.

p the pressure of the fluid, at the point M.

t the time counted from a given moment.

The gravity which acts upon the fluid at the point M may be decomposed into two forces; one perpendicular to the direction of the velocity, like the ordinate y, which will be $g \cos \gamma$; the other in the direction of the motion, which will be $g \sin \gamma$.* Thus the accelerating forces applied to the fluid are:

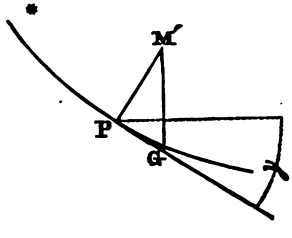
in the direction of the motion

$$g \sin \gamma - \frac{g\chi}{\omega} (av + bv^2)$$

and perpendicularly to the motion

$$g \cos \gamma.$$

We now remark that since the motion of the fluid is sensibly rectilinear, the particles which pass the point M move as if acted upon by a

* 

Angle $M' = \gamma$

$M'G = g$

$M'G$ is decomposed into

$M'P = g \cos \gamma$

and into

$GP = g \sin \gamma$.

Applying therefore the principles of the equilibrium of fluids, we shall have the two differential equations

$$\frac{dp}{ds} = g \sin \gamma - \frac{g\chi}{\omega}(av + bv^2) - \frac{dv}{dt}$$

$$\frac{dp}{dy} = -g \cos \gamma.$$

14. As γ is not a function of y we can integrate the latter equation, and shall have

$$p = -gy \cos \gamma + c,$$

c designating a quantity independent of y, though it may be a function of s. In order to determine its value, we may observe that, as the stream is open to the atmosphere, the pressure on its surface is a constant quantity. Let A be this constant pressure, and h the value of y, which corresponds to the surface of the water at the

The pressure $\frac{dp}{dy}$ and the force Y are in equilibrium; therefore,

$$\frac{dp}{dy} + Y = 0 \qquad \frac{dp}{dy} = -g \cos \gamma;$$

The pressure $\frac{dp}{ds}$ and the force S which produces the motion, taken in a contrary sense to its direction, are in equilibrium; therefore

$$\frac{dp}{ds} - S = 0, \text{ and } \frac{dp}{ds} = g \sin \gamma - \frac{g\chi}{\omega}(av + bv^2) - \frac{dv}{dt}.$$

section of the stream under consideration. In other words, let h be the distance of the point P on the axis from the surface of the stream in this section; we shall have

$$A = -gh\cos\gamma + c;$$

and eliminating c between this equation and the preceding, we have

$$p = A + g\,(h - y)\cos\gamma.$$

15. From this general formula we can now obtain a second value of $\frac{dp}{ds}$. Strictly speaking, the quantities h and $\cos\gamma$ are both functions of s; but, as the motion of the stream is very nearly rectilinear, the variations of the angle γ may be neglected when compared with the variations of s. In differentiating, therefore, we may consider as constant $\cos\gamma$, which moreover will always be very nearly unity. We have then

$$\frac{dp}{ds} = g\cos\gamma\,\frac{dh}{ds}.$$

16. Substituting this value of $\frac{dp}{ds}$ in the first member of the equation found above, we have

$$g\cos\gamma\,\frac{dh}{ds} = g\sin\gamma - \frac{g\chi}{\omega}(av + bv^2) - \frac{dv}{dt};$$

whence we deduce

$$\sin\gamma\,ds - \cos\gamma\,dh - \frac{\chi}{\omega}(av + bv^2)\,ds - \frac{dv\,ds}{g\,dt} = 0.$$

17. Calling Q the constant discharge in the unit of time, we have

$$v = \frac{Q}{\omega}, \text{ and thence } dv = -\frac{Q}{\omega^2} d\omega.$$

We have also evidently

$$\frac{ds}{dt} = v, \text{ or } \frac{ds}{dt} = \frac{Q}{\omega}; \text{ therefore } \frac{dv\,ds}{dt} = -\frac{Q^2}{\omega^3} d\omega.$$

Substituting this expression in the equation of Article 16, we have

$$\sin\gamma\, ds - \cos\gamma\, dh - \frac{\chi}{\omega}(av + bv^2)\, ds + \frac{Q^2}{g\,\omega^3} d\omega = 0.$$

18. Now, according to the hypotheses with which we set out (Art. 10), it is evident that we may choose for the axis of the stream any one of its linear elements, as for instance the line drawn lengthwise through the lowest points of the bed of the canal. In that case, the length s will be measured upon the bottom; the ordinate h will be the greatest depth of water measured perpendicularly to the axis, in the section which is normal to the end of the arc s; γ will be the angle, which the bottom makes with the horizon at that point; and, if we make $\sin\gamma = i$, the preceding equation becomes

$$i\, ds - \sqrt{1 - i^2}\, dh - \frac{\chi}{\omega}(av + bv^2)\, ds + \frac{Q^2}{g\,\omega^3} d\omega = 0.$$

The bed of the canal is supposed to be known; its inclination and profile may vary from one ex-

tremity to the other, provided it be by insensible degrees; i will be a known function of s; ω and χ will also be known functions of s and h; so will also v, since $v = \frac{Q}{\omega}$. The last equation therefore may be made to contain no other variable quantities than s, h, and their differentials. Consequently it becomes merely a problem of the integral calculus to determine the curve described by the fluid elements of the surface, which curve will be completely known when we obtain the value of the depth h, perpendicular to the axis, corresponding to any value of the length s.

19. When the inclination and the section of the canal are constant, the problem is much simplified. χ and ω will be functions of h only; and if we call x the variable width at the surface, this quantity will also be a function of h, and we shall have

$$d\omega = x\,dh.$$

20. Substituting this last expression in the equation of Article 18, and for $\frac{Q^2}{\omega^2}$ its value v^2, we have

$$ds = \frac{\frac{v^2 x}{g\omega} - \sqrt{1-i^2}}{\frac{\chi}{\omega}(av + bv^2) - i}\,dh,$$

an equation, which is of the form $ds = F(h)\,dh$, the simplest which we can have to inte-

grate. For each value of h this will give a corresponding value of $\frac{ds}{dh}$, the inverse of which $\frac{dh}{ds}$ will be the trigonometrical tangent of the angle, which the surface makes with the bottom of the bed; and since, by means of the angle i, we know the angle of the latter with the horizon, we can infer the angle which the surface makes with the horizon. This may sometimes be wanted.

21. Among the values which may be given to h in the preceding equation, there is one which is particularly remarkable. It is that which renders the quantity $\frac{ds}{dh}$ infinite. The sole condition to determine this value of h is, that it shall be such as to satisfy the equation

$$\frac{\chi}{\omega}(a v + b v^2) - i = 0.$$

The linear quantity $\frac{\omega}{\chi}$ is what is called the *mean radius* of the transverse section. In such practical cases as those we have been considering, when h increases, this quantity increases also, and consequently $\frac{\chi}{\omega}$ must diminish. Evidently also v, which is equal to $\frac{Q}{\omega}$, must diminish at the same time, and consequently $a v + b v^2$. From this it follows: (1.) That the above equation can only

be satisfied by a single value of h, which we call H; (2.) That according as we substitute in the polynome $\frac{\chi}{\omega}(a v + b v^2) - i$ a value of h greater or less than H, this polynome will be negative or positive.

22. The value H giving $\frac{ds}{dh} = \infty$ or $\frac{dh}{ds} = 0$, we see that, at the place where the stream has this depth, the surface has the same inclination as the bottom. This quantity H is therefore the height corresponding to a *uniform motion.* The preceding equation (Art. 21), which it satisfies, is in fact the equation given by M. de Prony, for the case of uniform motion.

23. Whenever, in such a canal as that mentioned in Article 19, we know that at a determined point of the length of the stream the depth h has a certain value h_0, different from that which belongs to a uniform motion, in order to find other points of the longitudinal profile, we have only to integrate the expression for ds given in Article 20, beginning with the limit h_0; but as even in the simplest cases to perform this integration rigorously would lead to very long calculations, it will be much better to give to the variable depth h a series of values $h_0, h_1, h_2, h_3, \ldots$ increasing or decreasing (according to the

question) by slight degrees from h_0 forward, and to integrate by approximation the differential ds successively from h_0 to h_1; from h_1 to h_2; from h_2 to h_3, ... until we reach the ordinate at which we think proper to stop. (See Art. 91 and following.)

24. We have said (Art. 22) that the equation $\frac{\chi}{\omega}(a\,v + b\,v^2) - i = 0$ is Prony's formula for the velocity of water in canals. Since i is the sine of the angle which the bed of the canal makes with the horizon it represents the same thing as I, and it is equal to the difference of level between the two extremities divided by the distance between them. Moreover $\frac{\omega}{\chi}$ is the mean radius which Prony calls R. The above equation becomes then

$$a\,v + b\,v^2 = RI,$$

which is its most common form.

25. Having obtained this simple equation to express the relation between the velocity, the section, and the inclination, it remains to determine numerically the values of the constant coefficients a and b. This can only be done experimentally. By a careful discussion of thirty-one experiments, chosen on account of the exactness with which they were made, Prony found that when the unit is the metre, the prop-

er values to give to these constants in order to agree with experiment are.

$$a = 0.0000444499$$
$$b = 0.0003093140.$$

According to Eytelwein, who determined them by means of a greater number of experiments, they are, the unit being the metre,

$$a = 0.0000242651$$
$$b = 0.0003655430.$$

26. As the English foot is our unit, we must see what alteration in the value of these constants is necessary, in order to give us a formula answering to our measures. The equation is

$$a\,v + b\,v^2 = RI.$$

I is a ratio, and will not change, whatever unit we adopt. If we had v and R given in feet, we might reduce them to metres, and, substituting their metrical value, the formula would hold good. We should thus have (see Appendix)

$$a\,\frac{v}{3.2808992} + b\,\frac{v^2}{(3.2808992)^2} = \frac{RI}{3.2808992},$$

a and b retaining their metrical values.

From this we deduce

$$a\,v + \frac{b}{3.2808992}\,v^2 = RI,$$

in which v, R, and I, are given in English feet. Consequently, in order to make the proper alteration, it is only necessary to divide the values of b by 3.2808992, and to retain for a its former numerical value. We thus have

1st, according to Prony

$$a = 0.0000444499$$
$$b = 0.0000942772;$$

2d, according to Eytelwein

$$a = 0.0000242651$$
$$b = 0.0001114155.$$

27. If we represent by Q the discharge per second, we shall have $v = \frac{Q}{\omega}$. Substituting this in the formula, and remembering that $R = \frac{\omega}{\chi}$, we shall have the equation

$$a\, Q\, \omega + b\, Q^2 = \frac{\omega^3}{\chi} I,$$

which gives directly the relation between ω, χ, I, and Q, in the case of uniform motion. Thus, any three of these quantities being determined *à priori*, the fourth must be such as to satisfy the conditions of this equation.

28. * Recapitulating the practical results which we have now obtained, we may say then, that

* This article contains all the results of the preceding investigation. In order to apply them

in open streams which are flowing with an uniform motion, calling

ω . . . the area of the section of the stream,

χ . . . that portion of the perimeter of the section of the bed, which is in contact with the water,

I . . . the fall divided by the length,

v . . . the mean velocity per second, $= \frac{Q}{\omega}$,

Q . . . the discharge per second,

R . . . the mean radius, or $\frac{\omega}{\chi}$,

(the unit being English feet,)

the relations between these several quantities may be expressed by the following formulæ,

$$0.0000444499\, v + 0.0000942772\, v^2 = RI,$$

whence we deduce,

$$v \quad -0.2357404865 + \sqrt{0\,0555735823 + 10607.020269\, RI}$$

and

$$0.0000444499\, Q\omega + 0.0000942772\, Q^2 = \frac{\omega^3}{\chi} I,$$

according to M. de Prony;

and

$$0.0000242651\, v + 0.0001114155\, v^2 = RI,$$

practically, we may take them as here given, without the necessity of recurring to the preceding 27 articles, which show the method by which these results may be obtained.

whence we deduce

$$v = -0.1088941604 + \sqrt{0.0118580490 + 8975.414285\ RI},$$

and

$$0.0000242651\,Q\,\omega + 0.0001114155\,Q^2 = \frac{\omega^3}{\chi} I,$$

according to Eytelwein. *

It is to be borne in mind that, to realize in practice what we call uniform motion, the canal or stream should be straight, and with the same section and inclination from one end to the other. In proportion as it varies from these conditions, we may expect to find the formulæ in fault.

The constant coefficients given by Prony were determined by thirty-one experiments; of which twenty-three were made with great care by Dubuat, upon small channels made purposely for the experiment; four in the Canal du Jard, and three in the River Hayne, likewise by Dubuat; and one by Chézy in the brook of Courpalet; the section varying from 1 to 312 square feet, and the fall per foot from 0.000028 to 0.047

* Logarithm of 0.0000444499 = 0.6478708 — 5.
Logarithm of 0.0000942772 = 0.9744666 — 5.
Logarithm of 10607.020269 = 4.0255934.
Logarithm of 0.0000242651 = 0.3849821 — 5.
Logarithm of 0.0001114155 = 0.0469455 — 4.
Logarithm of 8975.414285 = 3.9530545.

foot. Eytelwein's coefficients were determined by a greater number of experiments.

If we wish to apply the formulæ of uniform motion to a stream in which the inclination of the bed is not always the same, we should divide it into several parts, and apply them successively to each one of these.

29. The following table * has been calculated from the formula $a\,v + b\,v^2 = R\,I$, and will save some labor in applying it to practice. By

* M. de Prony has given tables of the values of $R\,I$ corresponding to those of v, from $v = 0.01$ metre to $v = 3$ metres. It may be useful to know how to apply them, supposing the quantities are given in feet. Suppose, then, $R\,I$ given in feet; I, being a ratio, does not change with the unit, and RI in feet, multiplied by 0.3047944, will be the value of RI expressed in metres. Thus the column of values of RI in M. de Prony's tables may be called a column of values of 0.3048 RI, the measures being English. Moreover the corresponding values of v in Prony's tables are in metres. But v metres $= (3.2808992\ v)$ feet. Therefore the values of v in the Table must be multiplied by 3.2808992 to give the answer in feet. Or if we retain the same numbers as in Prony's tables, we must call them values of $\frac{v}{3.281}$. Thus we can use his tables, by merely changing the caption, thus;

means of this table, if RI be given, we can find by inspection what will be the corresponding velocity; or, if the velocity be given, we can find the corresponding value of RI, whence we can determine at once the value of I if R be given, or that of R if I be given.

PRONY.

Values of v. — Values of RI corresponding to those of v.

ENGLISH TABLE.

Values of $\frac{v}{3.281}$. — Values of 0.3048 RI corresponding to those of $\frac{v}{3.281}$.

Supposing then RI given in feet; multiply it by 0.3048 and we have the value of 0.3048 RI, with which we enter the table. Opposite to it is the value of $\frac{v}{3.281}$; multiplying this by 3.281, we have the velocity v in feet.

TABLE

for calculating the MOTION *of* WATER *in* OPEN CANALS. *The equation for the motion being, according to* EYTELWEIN $0.0000242651\ v + 0.0001114155\ v^2 = RI$, *and according to* PRONY $0.0000444499\ v + 0.0000942772\ v^2 = RI$, *in which* v *is the mean velocity,* R *the mean radius, and the difference of level between the two extremities, divided by the length of the Canal. All in English feet.*

Velocity per second = v.	Value of $R\ I$ when velocity = v.		Velocity per second = v.	Value of $R\ I$ when velocity = v.	
	EYTELWEIN	PRONY		EYTELWEIN	PRONY
foot.			foot		
0.05	0.0000015	0.0000025	1.55	0.0003053	0.0002954
0.10	0.0000035	0.0000054	1.60	0.0003240	0.0003125
0.15	0.0000061	0.0000087	1.65	0.0003434	0.0003300
0.20	0.0000093	0.0000127	1.70	0.0003632	0.0003480
0.25	0.0000130	0.0000170	1.75	0.0003837	0.0003665
0.30	0.0000173	0.0000218	1.80	0.0004047	0.0003855
0.35	0.0000221	0.0000271	1.85	0.0004262	0.0004049
0.40	0.0000275	0.0000329	1.90	0.0004483	0.0004248
0.45	0.0000335	0.0000391	1.95	0.0004710	0.0004452
0.50	0.0000400	0.0000458	2.00	0.0004942	0.0004660
0.55	0.0000470	0.0000530	2.05	0.0005180	0.0004873
0.60	0.0000547	0.0000606	2.10	0.0005423	0.0005091
0.65	0.0000628	0.0000687	2.15	0.0005672	0.0005314
0.70	0.0000716	0.0000773	2.20	0.0005926	0.0005541
0.75	0.0000809	0.0000864	2.25	0.0006186	0.0005773
0.80	0.0000907	0.0000959	2.30	0.0006452	0.0006010
0.85	0.0001011	0.0001059	2.35	0.0006723	0.0006251
0.90	0.0001121	0.0001164	2.40	0.0007000	0.0006497
0.95	0.0001236	0.0001273	2.45	0.0007282	0.0006748
1.00	0.0001357	0.0001387	2.50	0.0007570	0.0007004
1.05	0.0001483	0.0001506	2.55	0.0007864	0.0007264
1.10	0.0001615	0.0001630	2.60	0.0008163	0.0007529
1.15	0.0001752	0.0001758	2.65	0.0008467	0.0007799
1.20	0.0001896	0.0001891	2.70	0.0008777	0.0008073
1.25	0.0002044	0.0002029	2.75	0.0009093	0.0008352
1.30	0.0002198	0.0002171	2.80	0.0009414	0.0008636
1.35	0.0002358	0.0002318	2.85	0.0009741	0.0008925
1.40	0.0002523	0.0002470	2.90	0.0010074	0.0009218
1.45	0.0002694	0.0002627	2.95	0.0010412	0.0009516
1.50	0.0002871	0.0002788	3.00	0.0010755	0.0009818

TABLE — *continued.*

Velocity per second = v.	Value of RI when velocity = v.		Velocity per second = v.	Value of RI when velocity = v.	
	Eytelwein	Prony		Eytelwein	Prony
foot			foot		
3.05	0.0011105	0.0010126	5.15	0.0030800	0.0027294
3.10	0.0011459	0.0010438	5.20	0.0031389	0.0027804
3.15	0.0011820	0.0010755	5.25	0.0031983	0.0028319
3.20	0.0012185	0.0011076	5.30	0.0032583	0.0028838
3.25	0.0012557	0.0011403	5.35	0.0033188	0.0029363
3.30	0.0012934	0.0011734	5.40	0.0033799	0.0029891
3.35	0.0013317	0.0012069	5.45	0.0034416	0.0030425
3.40	0.0013705	0.0012410	5.50	0.0035038	0.0030964
3.45	0.0014099	0.0012755	5.55	0.0035665	0.0031507
3.50	0.0014498	0.0013105	5.60	0.0036299	0.0032055
3.55	0.0014903	0.0013459	5.65	0.0036938	0.0032607
3.60	0.0015313	0.0013819	5.70	0.0037582	0.0033164
3.65	0.0015729	0.0014183	5.75	0.0038232	0.0033726
3.70	0.0016151	0.0014551	5.80	0.0038888	0.0034293
3.75	0.0016578	0.0014925	5.85	0.0039549	0.0034864
3.80	0.0017011	0.0015303	5.90	0.0040216	0.0035440
3.85	0.0017449	0.0015686	5.95	0.0040888	0.0036021
3.90	0.0017893	0.0016073	6.00	0.0041565	0.0036607
3.95	0.0018342	0.0016465	6.05	0.0042249	0.0037197
4.00	0.0018798	0.0016862	6.10	0.0042938	0.0037792
4.05	0.0019258	0.0017264	6.15	0.0043632	0.0038392
4.10	0.0019724	0.0017670	6.20	0.0044333	0.0038996
4.15	0.0020196	0.0018082	6.25	0.0045038	0.0039605
4.20	0.0020673	0.0018497	6.30	0.0045750	0.0040219
4.25	0.0021156	0.0018918	6.35	0.0046467	0.0040837
4.30	0.0021644	0.0019343	6.40	0.0047189	0.0041461
4.35	0.0022138	0.0019773	6.45	0.0047917	0.0042089
4.40	0.0022638	0.0020208	6.50	0.0048650	0.0042721
4.45	0.0023143	0.0020647	6.55	0.0049389	0.0043359
4.50	0.0023654	0.0021091	6.60	0.0050134	0.0044001
4.55	0.0024170	0.0021540	6.65	0.0050884	0.0044648
4.60	0.0024692	0.0021994	6.70	0.0051640	0.0045299
4.65	0.0025219	0.0022452	6.75	0.0052402	0.0045955
4.70	0.0025752	0.0022915	6.80	0.0053169	0.0046616
4.75	0.0026291	0.0023383	6.85	0.0053941	0.0047282
4.80	0.0026835	0.0023855	6.90	0.0054719	0.0047952
4.85	0.0027385	0.0024332	6.95	0.0055503	0.0048627
4.90	0.0027940	0.0024814	7.00	0.0056292	0.0049307
4.95	0.0028501	0.0025301	7.05	0.0057087	0.0049992
5.00	0.0029067	0.0025792	7.10	0.0057887	0.0050681
5.05	0.0029639	0.0026288	7.15	0.0058693	0.0051375
5.10	0.0030217	0.0026788	7.20	0.0059505	0.0052074

TABLE — *continued.*

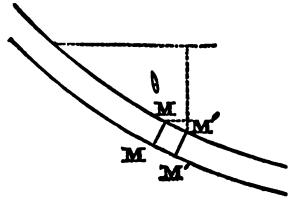
Velocity per second = v.	Value of RI when velocity = v.		Velocity per second = v.	Value of RI when velocity = v.	
	EYTELWEIN	PRONY		EYTELWEIN	PRONY
foot			foot		
7.25	0.0060322	0.0052777	8.65	0.0085463	0.0074385
7.30	0.0061145	0.0053485	8.70	0.0086442	0.0075226
7.35	0.0061973	0.0054198	8.75	0.0087426	0.0076070
7.40	0.0062807	0.0054915	8.80	0.0088416	0.0076920
7.45	0.0063646	0.0055638	8.85	0.0089411	0.0077774
7.50	0.0064491	0.0056365	8.90	0.0090412	0.0078633
7.55	0.0065342	0.0057096	8.95	0.0091419	0.0079497
7.60	0.0066198	0.0057833	9.00	0.0092430	0.0080365
7.65	0.0067060	0.0058574	9.05	0.0093448	0.0081238
7.70	0.0067927	0.0059319	9.10	0.0094471	0.0082116
7.75	0.0068800	0.0060070	9.15	0.0095500	0.0082998
7.80	0.0069678	0.0060825	9.20	0.0096535	0.0083886
7.85	0.0070562	0.0061585	9.25	0.0097575	0.0084778
7.90	0.0071452	0.0062350	9.30	0.0098620	0.0085674
7.95	0.0072347	0.0063119	9.35	0.0099671	0.0086576
8.00	0.0073247	0.0063893	9.40	0.0100728	0.0087482
8.05	0.0074153	0.0064672	9.45	0.0101790	0.0088392
8.10	0.0075065	0.0065456	9.50	0.0102857	0.0089308
8.15	0.0075983	0.0066244	9.55	0.0103931	0.0090228
8.20	0.0076906	0.0067037	9.60	0.0105010	0.0091153
8.25	0.0077834	0.0067835	9.65	0.0106094	0.0092083
8.30	0.0078768	0.0068637	9.70	0.0107184	0.0093017
8.35	0.0079708	0.0069444	9.75	0.0108280	0.0093956
8.40	0.0080653	0.0070256	9.80	0.0109381	0.0094900
8.45	0.0081604	0.0071072	9.85	0.0110488	0.0095848
8.50	0.0082560	0.0071893	9.90	0.0111601	0.0096802
8.55	0.0083522	0.0072719	9.95	0.0112719	0.0097760
8.60	0.0084490	0.0073550	10.00	0.0113842	0.0098722

CHAPTER II.

THEORY OF THE MOTION OF WATER IN PIPES.

30. The conditions for the uniform motion may be easily realized in the case of pipes, and therefore the formula which belongs to that motion may be used with confidence in practice. The following method of arriving at this formula has been given by Bélanger.

Let $MMM'M'$ be an infinitely thin section of the stream. As the motion is permanent and uniform, the forces which act upon it must be in equilibrium.



If we call p the mean pressure upon the section MM, that upon the section $M'M'$ will be $p + dp$; we must therefore take into account the force $\omega\, dp$, which acts in a direction contrary to the motion, ω being the area of the section. The resistance of the pipe is also another retarding force; this resistance is a function of the velocity, and also proportional to the surface in contact with the element under consideration. Calling v

the velocity; χ the circumference of the pipe; s the length measured from some fixed point, the resistance will be

$$\chi \varphi (v)\, ds,$$

$\varphi (v)$ denoting an unknown function of the velocity.

The force which acts upon the mass $MMM'M'$ in the direction of the motion is, its weight decomposed in the direction of s. It is therefore $\omega\, ds\, g \cos \theta$, calling θ the angle which the tangent at the point in question makes with the vertical, and g being the accelerating force of gravity.

Calling dz the difference of level between the points M and M', we have

$dz = ds \cos \theta$; hence $\omega\, ds\, g \cos \theta = \omega g\, dz$.

Putting this force in equilibrium with the two preceding, we have

$$\omega g\, dz = \omega\, dp + \chi \varphi (v)\, ds.$$

31. If we now integrate from the origin of the pipe where we suppose the pressure to be P to the section MM, we have

$$\omega g z = \omega (p - P) + \chi \varphi (v)\, s.$$

Calling the pressure at the end of the pipe Π, the whole length λ, and the difference of level between the two extremities ζ, we shall have, integrating from one end to the other,

$$\omega g \zeta = \omega (\Pi - P) + \chi \varphi (v)\, \lambda.$$

Eliminating $\varphi\ (v)$ between these two equations, we shall have

$$g\ (\lambda z - s\zeta) = \lambda p - (\lambda - s)\ P - s\ \Pi,$$

which is the same equation as that given by Prony in Article 119 of the *Physico-Mathematical Researches*. By means of it we can determine the pressure p, which generally varies from one point to another of the pipe.

32. By means of the equation

$$\omega g \zeta = \omega\ (\Pi - P) + \chi \varphi\ (v)\ \lambda$$

we shall be able to explain all the circumstances of the motion. We have already seen that experiment has proved, that the resistance caused by the friction against the sides may be represented by the expression

$$\chi\ (a v + b v^2)\ d s.$$

Consequently

$$\varphi\ (v) = a v + b v^2,$$

a and b being two constant numbers.

Substituting this in the last equation, we shall have

$$a v + b v^2 = \frac{\omega}{\chi} \frac{g \zeta - (\Pi - P)}{\lambda}.$$

33. If the pipe have a diameter D, we have

$$\frac{\omega}{\chi} = \tfrac{1}{4} D;$$

therefore

$$a v + b v^2 = \tfrac{1}{4} D \frac{g \zeta - (\Pi - P)}{\lambda}.$$

34. We have represented by P the pressure upon the upper extremity of the pipe. And this pressure is produced by the weight of the column above it.

But the weight of this column not only exerts a pressure, but imparts a certain velocity to the fluid beneath it, so that, strictly speaking, we cannot in the formula above obtained substitute this weight of water for the pressure alone. But in the case of small velocities, the difference is insensible, and there will not result any perceptible error if we make $P = g H$, calling H the column upon the upper extremity, and $\Pi = g H'$, H' being the column upon the lower. By making these substitutions we obtain

$$a v + b v^2 = \tfrac{1}{4} D g \frac{\zeta - H' + H}{\lambda}.$$

35. If we divide both members of this equation by g, and make $\frac{\zeta - H' + H}{\lambda} = j$, we shall have, making $\frac{a}{g} = \alpha$ and $\frac{b}{g} = \beta$,

$$\alpha v + \beta v^2 = \tfrac{1}{4} D j,$$

which is the form under which it is most commonly given. M. de Prony has determined, by

means of fifty-one experiments made upon pipes by Couplet, Bossut, and Dubuat, the numerical values of the coefficients α and β. They are

$$\alpha = 0.0000173314\,; \quad \beta = 0.0003482590\,;$$

the unit being the metre.

To find the proper values to give them when we take the English foot for our measure, we observe that if v, D, and j, were given in feet, we might reduce them to metres by dividing their values by 3.2808992, and the formula would hold good. We should thus have

$$\frac{\alpha v}{3.2808992} + \frac{\beta v^2}{(3.2808992)^2} = \tfrac{1}{4}\,\frac{D}{3.2808992}\,j,$$

α and β keeping the numerical values which Prony gives them. From this we deduce

$$\alpha v + \frac{\beta}{3.2808992}\,v^2 = \tfrac{1}{4}\,D j.$$

Thus the constant α will retain its former value, but β must be divided by 3.2808992. We shall have then

$\alpha = 0.0000173314$ log of $\alpha = 0.2388336 - 5$
$\beta = 0.0001061473$ log of $\beta = 0.0259093 - 4$

and the equation becomes

$$0.0000173314\,v + 0.0001061473\,v^2 = \tfrac{1}{4}\,D\,j,$$

the unit being the English foot.

36. According to M. de Prony, we may, except in cases where the velocity is exceedingly small, neglect the term containing only the first power of v, in the equation of Article 35. This equation then becomes

$$0.0001061473\, v^2 = \tfrac{1}{4}\, Dj,$$

whence we deduce the very simple expression

$$v = 48.53\sqrt{Dj};$$

the logarithm of 48.53 being 1.6860154.

37. In using this coefficient we commit an error by excess; and it is worth while to see how great this error may be, within the common limits of practice. If we make $v = c\sqrt{Dj}$, we can easily find by means of a table (Art. 44) calculated from the formula of Article 35, what should be the value of c corresponding to various velocities, in order to make this simple formula give the same results as the rigorous formula. We shall find that when

$v =$	0.50	1	2	3	4	5
$c =$	42.137	44.992	46.661	47.262	47.569	47.758

6	7	8	9	10
47.883	47.975	48.043	48.096	48.139.

Hence if, after using the coefficient 48.53, we find the value of v equal to 3 feet for instance, it

would be more correct to use the formula $47.262\sqrt{Dj}$, and calculate a second time. Our result would more nearly agree with that which the rigorous formula would give.

38. If we represent by Q the discharge per second, it is evident that, the diameter being D, we have

$$Q = \frac{\pi D^2}{4} v, \quad \text{whence } v = \frac{4Q}{\pi D^2},$$

π being equal to 3.1415926.

Substituting this value of v in the formula of Article 35, it becomes

$$\alpha \frac{4Q}{\pi D^2} + \beta \frac{16 Q^2}{\pi^2 D^4} = \tfrac{1}{4} Dj;$$

whence we deduce

$$0.000088268\, D^2 Q + 0.000688319\, Q^2 = D^5 j;$$

by means of which, knowing the value of j and of the diameter, we find at once the discharge.

39. The formula of Article 36 becomes

$$\frac{4Q}{\pi D^2} = 48.53\sqrt{Dj},$$

whence we deduce

$$Q = 38.116\sqrt{D^5 j}.$$

From what was said in Article 37 it is evident that this value of the coefficient will generally be a little too large; since at the various veloci-

ties we ought strictly to have used, instead of 48.53, the corresponding values of the coefficients for those velocities, given in that article. We should thus have

v	=	0.50	1	2	3	4
coefficient	=	33.094	35.337	36.648	37.120	37.360

5	6	7	8	9	10
37.509	37.607	37.680	37.733	37.775	37.808;

and thus, if we had first ascertained that the velocity was about 5 feet, instead of

$$Q = 38.116\sqrt{D^5 j},$$

we might use more correctly $Q = 37.509\sqrt{D^5 j}$.

40. Eytelwein has given the following formula for the uniform motion of water in pipes;

$$v = \frac{-0.03988 + \sqrt{0.0015904 + 891.85\,\frac{D\zeta}{\lambda}}}{1 + 68.865\,\frac{D}{\lambda}},$$

the unit being the metre.

This formula becomes *

* If D, ζ, and λ are given in feet, we may multiply them by 0.30479449 to reduce them to metres and enter them as such in the formula. The result will be a velocity in metres, which we must

$$v = \frac{-0.130842 + \sqrt{0.0171955 + 2926.0664 \frac{D\zeta}{\lambda}}}{1 + 68.865 \frac{D}{\lambda}},$$

the unit being the English foot.

41. Eytelwein has also given a simpler formula, which will be sufficiently correct in practice. It is

$v = 26.44 \sqrt{\frac{D\zeta}{\lambda + 54D}}$, the unit being the metre.

or

$v = 47.895 \sqrt{\frac{D\zeta}{\lambda + 54D}}$, the unit being the foot,

and the logarithm of 47.895 being 1.6802583.

multiply by 3.2808992 to reduce to feet. Thus

$$v = 3.2808992 \frac{-0.03988 + \sqrt{0.0015904 + 0.3048 \times 891.85 \frac{D\zeta}{\lambda}}}{1 + 68.865 \frac{D}{\lambda}}$$

will be the formula in English feet. In like manner the simple formula is

$$v = \frac{26.44}{\sqrt{0.30479449}} \sqrt{\frac{D\zeta}{\lambda + 54D}}$$

in English feet.

42. Substituting in the last equation $\frac{4}{\pi}\frac{Q}{D^2}$ for v, we have

$$Q = \frac{\pi}{4} 47.895 \sqrt{\frac{D^5 \zeta}{\lambda + 54 D}}$$

or

$$Q = 37.614 \sqrt{\frac{D^5 \zeta}{\lambda + 54 D}},$$

the logarithm of 37.614 being 1.5753482.

43. * Recapitulating the results which we have now obtained, we may say then, that, calling

D . . . the diameter of the pipe,

ζ . . . the difference of level between the upper surface of the basin where the pipe commences and the upper surface of the basin where it discharges, or the extremity of the pipe if it discharges in the open air,

λ . . . the length of the pipe,

j . . . $\frac{\zeta}{\lambda}$,

v . . . the mean velocity per second or the discharge divided by the section,

* This article contains all the formulæ on the Motion of Water in Pipes, which are needed in application. The preceding articles of this chapter need only be referred to, in order to see the process of investigation, and to verify the formulæ here given.

Q . . . the discharge per second,
and the English foot being the unit,
we have between these several quantities the following relations ;

$$0.0000173314\ v + 0.0001061473\ v^2 = \tfrac{1}{4}\ Dj,$$

whence we deduce

$$v = -\ 0.0816383 + \sqrt{0.00666482 + 2355.2158\ Dj}$$

and

$$0.000088268\ D^2Q + 0.0006881319\ Q^2 = D^5j;$$

from which may be deduced as approximate formulæ, sufficiently exact for practice,

$$v = 48.53\sqrt{Dj}$$

and

$$Q = 38.116\sqrt{D^5j}$$

according to M. de Prony.
Also

$$v = \frac{-0.130842 + \sqrt{0.0171955 + 2926.0664\ \frac{D\zeta}{\lambda}}}{1 + 68.865\ \frac{D}{\lambda}},$$

and, as approximations sufficiently exact for practice,

$$v = 47.895\sqrt{\frac{D\zeta}{\lambda + 54\ D}}$$

$$Q = 37.614 \sqrt{\frac{D^5 \zeta}{\lambda + 54\ D}}$$

according to Eytelwein.*

The above formulæ suppose that the pipe is straight, of the same diameter throughout, and that the level of the basin whence it starts and of that in which it discharges are constant or sensibly so.

Prony's coefficients were determined by fifty-one experiments made upon pipes whose diameter varied from one inch to nearly twenty, and whose length was from 10 feet to 7500.

44. To facilitate the application of these formulæ the following tables have been calculated. The first is from the first formula of Prony. It gives the value of the velocity when $\frac{1}{4}\ Dj$ is known, or the value of $\frac{1}{4}\ Dj$ when the velocity is known. Knowing $\frac{1}{4}\ Dj$ we can assume a value for either of the quantities D or j, and thus

* Logarithm of 0.0000173314 = 0.2388336 — 5.
" 0.0001061473 = 0.0259093 — 4.
" 2355.2158 = 3.3720307.
" 0.000088268 = 0.9543037 — 5.
" 0.0006881319 = 0.8377895 — 4.
" 48.53 = 1.6860154.
" 38.116 = 1.5811065.
" 47.895 = 1.6802583.
" 37.614 = 1.5753482.

determine the other. The second table is calculated from Prony's approximate formula for the discharge. It gives the discharge per second when $D^5 j$ is known, and the value of D or j when either of them and the discharge are known.*

* It may be useful to know how to apply the French tables published by M. de Prony, in which the metre is the unit. By the same reasoning as that we used in the note to Article 29, we shall see that, if our quantities are all given in English feet, we have only to call Prony's column of values of v, values of $\frac{v}{3.2808992}$; and his column of values of $\frac{1}{4} D j$, values of 0.3048 $\frac{1}{4} D j$ or 0.0762 $D j$. Thus $D j$ being given in feet, we multiply it by 0.0762 and look for the product in Prony's column of values of $\frac{1}{4} D j$. The corresponding value of v, being multiplied by 3.281, will give the velocity in feet.

TABLE

for finding the VELOCITY *of* WATER *in* PIPES, *calculated from* PRONY'S *exact formula* 0.0000173314 v + 0.000106147 v^2 = $\frac{1}{4}$ Dj. *In which* v *represents the mean velocity per second in English feet,* D *the diameter of the pipe,* j *the head of water above the discharging orifice divided by the length of the pipe; all in English feet.*

Velocity = *v*.	Value of $\frac{1}{4}$ *Dj* when velocity = *v*.	Velocity = *v*.	Value of $\frac{1}{4}$ *Dj* when velocity = *v*.	Velocity = *v*.	Value of $\frac{1}{4}$ *Dj* when velocity = *v*.	Velocity = *v*.	Value of $\frac{1}{4}$ *Dj* when velocity = *v*.
foot.		foot.		foot.		foot.	
0.05	0.0000011	1.65	0.0003176	3.25	0.0011775	4.85	0.0025809
0.10	0.0000026	1.70	0.0003362	3.30	0.0012131	4.90	0.0026335
0.15	0.0000050	1.75	0.0003554	3.35	0.0012495	4.95	0.0026867
0.20	0.0000077	1.80	0.0003751	3.40	0.0012866	5.00	0.0027403
0.25	0.0000110	1.85	0.0003954	3.45	0.0013243	5.05	0.0027945
0.30	0.0000148	1.90	0.0004161	3.50	0.0013610	5.10	0.0028492
0.35	0.0000191	1.95	0.0004374	3.55	0.0013995	5.15	0.0029045
0.40	0.0000239	2.00	0.0004593	3.60	0.0014381	5.20	0.0029603
0.45	0.0000293	2.05	0.0004816	3.65	0.0014774	5.25	0.0030166
0.50	0.0000352	2.10	0.0005045	3.70	0.0015172	5.30	0.0030735
0.55	0.0000416	2.15	0.0005279	3.75	0.0015577	5.35	0.0031309
0.60	0.0000486	2.20	0.0005519	3.80	0.0015986	5.40	0.0031888
0.65	0.0000561	2.25	0.0005764	3.85	0.0016401	5.45	0.0032472
0.70	0.0000641	2.30	0.0006014	3.90	0.0016821	5.50	0.0033063
0.75	0.0000727	2.35	0.0006269	3.95	0.0017246	5.55	0.0033658
0.80	0.0000818	2.40	0.0006530	4.00	0.0017677	5.60	0.0034258
0.85	0.0000914	2.45	0.0006796	4.05	0.0018113	5.65	0.0034864
0.90	0.0001016	2.50	0.0007067	4.10	0.0018554	5.70	0.0035475
0.95	0.0001123	2.55	0.0007344	4.15	0.0019000	5.75	0.0036091
1.00	0.0001235	2.60	0.0007627	4.20	0.0019452	5.80	0.0036713
1.05	0.0001352	2.65	0.0007913	4.25	0.0019909	5.85	0.0037340
1.10	0.0001473	2.70	0.0008206	4.30	0.0020372	5.90	0.0037972
1.15	0.0001603	2.75	0.0008504	4.35	0.0020840	5.95	0.0038609
1.20	0.0001736	2.80	0.0008807	4.40	0.0021313	6.00	0.0039253
1.25	0.0001875	2.85	0.0009116	4.45	0.0021791	6.05	0.0039901
1.30	0.0002019	2.90	0.0009430	4.50	0.0022275	6.10	0.0040555
1.35	0.0002168	2.95	0.0009749	4.55	0.0022764	6.15	0.0041213
1.40	0.0002323	3.00	0.0010073	4.60	0.0023258	6.20	0.0041877
1.45	0.0002483	3.05	0.0010403	4.65	0.0023758	6.25	0.0042547
1.50	0.0002648	3.10	0.0010738	4.70	0.0024263	6.30	0.0043221
1.55	0.0002819	3.15	0.0011078	4.75	0.0024773	6.35	0.0043901
1.60	0.0002995	3.20	0.0011424	4.80	0.0025288	6.40	0.0044587

TABLE — *continued.*

Velocity = v.	Value of $\frac{1}{4} Dj$ when velocity = v.	Velocity = v.	Value of $\frac{1}{4} Dj$ when velocity = v.	Velocity = v.	Value of $\frac{1}{4} Dj$ when velocity = v.	Velocity = v.	Value of $\frac{1}{4} Dj$ when velocity = v.
foot.		foot.		foot.		foot.	
6.45	0.0045277	7.35	0.0058617	8.25	0.0073676	9.15	0.0090455
6.50	0.0045974	7.40	0.0059408	8.30	0.0074563	9 20	0.0091437
6.55	0.0046675	7.45	0.0060205	8.35	0.0075455	9.25	0 0092425
6.60	0.0047382	7.50	0.0061008	8.40	0.0076353	9.30	0.0093418
6.65	0.0048094	7.55	0.0061815	8.45	0.0077256	9.35	0.0094417
6.70	0.0048811	7.60	0.0062628	8.50	0.0078165	9.40	0.0095420
6.75	0.0049533	7.65	0.0063446	8.55	0.0079078	9.45	0.0096429
6.80	0.0050261	7.70	0.0064269	8.60	0.0079997	9.50	0.0097445
6.85	0.0050994	7.75	0.0065098	8.65	0.0080921	9.55	0.0098464
6.90	0.0051732	7.80	0.0065931	8.70	0.0081850	9.60	0.0099489
6.95	0.0052476	7.85	0.0066771	8.75	0.0082785	9.65	0.0100519
7.00	0.0053225	7.90	6.0067615	8.80	0.0083725	9.70	0.0101555
7.05	0.0053980	7.95	0.0068465	8.85	0.0084671	9.75	0.0102596
7.10	0.0054739	8.00	0.0069321	8.90	0.0085621	9.80	0.0103642
7.15	0.0055504	8.05	0.0070181	8.95	0.0086577	9.85	0.0104693
7.20	0.0056274	8.10	0.0071047	9.00	0.0087539	9.90	0.0105750
7.25	0.0057050	8.15	0.0071918	9.05	0.0088506	9.95	0.0106812
7.30	0.0057831	8.20	0.0072794	9.10	0.0089478	10.00	0.0107880

TABLE

giving the QUANTITY DISCHARGED *by* WATER-PIPES, *calculated from the formula* $Q = 38.116 \sqrt{D^5 j}$, *in which* Q *is the discharge per second in English cubic feet,* D *the diameter,* j *the head divided by the length of pipe.*

Q	$D^5 j$	Q	$D^5 j$	Q	$D^5 j$	Q	$D^5 j$
0.05	0.0000017	1.85	0.0023558	3.65	0.0091701	5.45	0.0204448
0.10	0.0000069	1.90	0.0024848	3.70	0.0094231	5.50	0.0208216
0.15	0.0000155	1.95	0.0026173	3.75	0.0096795	5.55	0.0212019
0.20	0.0000275	2.00	0.0027533	3.80	0.0099393	5.60	0.0215857
0.25	0.0000430	2.05	0.0028927	3.85	0.0102026	5.65	0.0219728
0.30	0.0000619	2.10	0.0030355	3.90	0.0104693	5.70	0.0223634
0.35	0.0000843	2.15	0.0031818	3.95	0.0107395	5.75	0.0227575
0.40	0.0001101	2.20	0.0033315	4.00	0.0110131	5.80	0.0231550
0.45	0.0001394	2.25	0.0034846	4.05	0.0112901	5.85	0.0235560
0.50	0.0001721	2.30	0.0036412	4.10	0.0115706	5.90	0.0239603
0.55	0.0002082	2.35	0.0038012	4.15	0.0118546	5.95	0.0243682
0.60	0.0002478	2.40	0.0039647	4.20	0.0121419	6.00	0.0247794
0.65	0.0002908	2.45	0.0041316	4.25	0.0124327	6.05	0.0251942
0.70	0.0003372	2.50	0.0043020	4.30	0.0127270	6.10	0.0256123
0.75	0.0003872	2.55	0.0044758	4.35	0.0130247	6.15	0.0260339
0.80	0.0004405	2.60	0.0046530	4.40	0.0133258	6.20	0.0264590
0.85	0.0004973	2.65	0.0048337	4.45	0.0136304	6.25	0.0268874
0.90	0.0005575	2.70	0.0050178	4.50	0.0139384	6.30	0.0273193
0.95	0.0006212	2.75	0.0052054	4.55	0.0142499	6.35	0.0277547
1.00	0.0006883	2.80	0.0053964	4.60	0.0145648	6.40	0.0281935
1.05	0.0007589	2.85	0.0055908	4.65	0.0148831	6.45	0.0286357
1.10	0.0008329	2.90	0.0057888	4.70	0.0152049	6.50	0.0290814
1.15	0.0009103	2.95	0.0059901	4.75	0.0155302	6.55	0.0295306
1.20	0.0009912	3.00	0.0061949	4.80	0.0158588	6.60	0.0299831
1.25	0.0010755	3.05	0.0064031	4.85	0.0161910	6.65	0.0304391
1.30	0.0011633	3.10	0.0066147	4.90	0.0165265	6.70	0.0308985
1.35	0.0012545	3.15	0.0068298	4.95	0.0168655	6.75	0.0313615
1.40	0.0013491	3.20	0.0070484	5.00	0.0172080	6.80	0.0318278
1.45	0.0014472	3.25	0.0072703	5.05	0.0175538	6.85	0.0322976
1.50	0.0015487	3.30	0.0074958	5.10	0.0179031	6.90	0.0327708
1.55	0.0016537	3.35	0.0077246	5.15	0.0182559	6.95	0.0332475
1.60	0.0017621	3.40	0.0079570	5.20	0.0186121	7.00	0.0337276
1.65	0.0018739	3.45	0.0081927	5.25	0.0189718	7.05	0.0342111
1.70	0.0019892	3.50	0.0084319	5.30	0.0193348	7.10	0.0346981
1.75	0.0021080	3.55	0.0086745	5.35	0.0197014	7.15	0.0351885
1.80	0.0022301	3.60	0.0089206	5.40	0.0200714	7.20	0.0356824

TABLE — *continued.*

Q	$D^5 j$	Q	$D^5 j$	Q	$D^5 j$	Q	$D^5 j$
7.25	0.0361797	7.95	0.0435034	8.65	0.0515017	9.35	0.0601745
7.30	0.0366805	8.00	0.0440523	8.70	0.0520988	9.40	0.0608198
7.35	0.0371846	8.05	0.0446047	8.75	0.0526993	9.45	0.0614685
7.40	0.0376923	8.10	0.0451605	8.80	0.0533033	9.50	0.0621207
7.45	0.0382034	8.15	0.0457198	8.85	0.0539108	9.55	0.0627763
7.50	0.0387179	8.20	0.0462825	8.90	0.0545217	9.60	0.0634354
7.55	0.0392358	8.25	0.0468486	8.95	0.0551360	9.65	0.0640979
7.60	0.0397572	8.30	0.0474182	9.00	0.0557538	9.70	0.0647638
7.65	0.0402820	8.35	0.0479912	9.05	0.0563750	9.75	0.0654332
7.70	0.0408104	8.40	0.0485677	9.10	0.0569996	9.80	0.0661060
7.75	0.0413421	8.45	0.0491476	9.15	0.0576277	9.85	0.0667823
7.80	0.0418772	8.50	0.0497310	9.20	0.0582592	9.90	0.0674620
7.85	0.0424159	8.55	0.0503178	9.25	0.0588942	9.95	0.0681452
7.90	0.0429579	8.60	0.0509080	9.30	0.0595326	10.00	0.0688318

45. We shall now proceed to consider the effect of various obstructions to the free passage of water in pipes, in order to estimate their influence upon the discharge. The effect of these obstructions may, in general, be measured by a certain head requisite to overcome them. When therefore they occur, we must, before applying the formulæ found in the preceding pages, deduct from the real head a certain height corresponding to the number and character of these obstructions, and sufficient to counteract their effects.

46. Whenever there occurs in a pipe any sudden contraction, the water in passing through the contracted section must necessarily move with a greater velocity. For, the discharge being constant, the same quantity must pass each sec-

tion in the same time; consequently, when the section is less, the velocity must be greater. This excess of velocity must be produced by the head of water, which causes the general motion, and will consequently absorb a part of the moving force.

To determine the expression of this resistance, let v, v' represent the velocities in the two parts where the diameters are different, and h, h' the heights *due* to those velocities, or the heights through which a heavy body must fall to acquire them; we have

$$h = \frac{v^2}{2g} \qquad h' = \frac{v'^2}{2g}$$

(see Articles 57 and 60.) The difference

$$h' - h = \frac{v'^2 - v^2}{2g}$$

will be the measure of the head consumed in producing this excess of velocity. Calling D, D' the diameters of the two parts respectively, and remembering that the discharge Q is constant, we have

$$v = \frac{4Q}{\pi D^2} \qquad v' = \frac{4Q}{\pi D'^2},$$

whence

$$h' - h = \frac{16 Q^2}{2\pi^2 g}\left(\frac{1}{D^4} - \frac{1}{D'^4}\right),$$

$\pi = 3.1415926.$

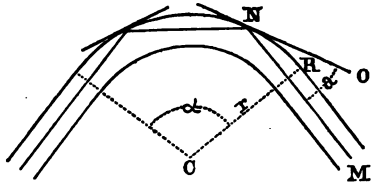
Moreover, whenever the water which is flowing in a pipe is forced to pass through an aperture smaller than the diameter, there occurs a contraction in the fluid vein (see Art. 58), which diminishes the section still more, and consequently augments the effect of the obstruction. This last circumstance is taken into account by multiplying the section of the contracted portion of the pipe by a coefficient m, whose value is deduced from experiment in various cases.

Thus we shall have $v' = \frac{4\,Q}{\pi\, m\, D^2}$; and for that portion of the head wasted in overcoming the obstruction

$$h' - h = \frac{16\,Q^2}{2\,\pi^2\,g}\left(\frac{1}{m^2\,D'^4} - \frac{1}{D^4}\right).$$

If the contraction be caused by a diaphragm having an opening of the diameter D', we have $m = 0.62$; if by a piece of pipe smaller than the rest, $m = 0.82$; if between the two pipes of different diameters there is a conical portion, having nearly the shape of the contracted vein, $m = 0.90$. All unnecessary enlargements should also be avoided; for wherever they occur there must be a retardation of the velocity, since the same quantity must pass through every section in the unit of time, and therefore a portion of the head must be consumed in restoring to the current its former velocity, after it has passed the enlarged part.

47. All changes of direction are a great obstruction to the free passage of water in pipes, and the more sudden they are, the more disadvantageous their effect. The resistance occasioned by this cause may likewise be measured by a certain head of water employed to overcome it. From a number of experiments made with a view to this purpose, Dubuat ascertained that it might be represented by the expression $\frac{v^2 s^2 n}{m}$, in which v represents the mean velocity, s the sine of the angle of incidence or of reflexion MNO, n the number of reflexions, and m a constant quantity.



To find the value of the angle of reflexion, we may observe that the versed sine of this angle is equal to the half diameter of the pipe, divided by the exterior radius of curvature CR of the bend. Therefore D being the diameter, and r the radius of curvature,

$$\text{cosine of angle of reflexion} = 1 - \frac{D}{2r}.$$

To find the number of reflexions in any curve, a being the angle at centre and α the angle of reflexion,

$$n = \frac{a}{2\alpha}.$$

If n is not a whole number, the last reflexion will throw the water in an irregular manner against the sides, and not according to the axis of the pipe.

The constant m, Dubuat determined to be 2998.50, when the French inch is the unit. Taking for our unit the English foot, we shall have $m = 266.3117$, and the formula becomes

$$\frac{v^2 s^2 n}{266.3117} \text{ or } 0.003755\ v^2 s^2 n.$$

This quantity represents in English feet the head requisite to overcome the resistance of the bend.*

48. There are other causes also which oppose the motion of water in pipes. The air, which fills them before the water is admitted, or that which enters with it at the orifice, rises and occupies the most elevated parts of the pipe. Being there condensed, it acquires sufficient force to impede greatly, or even entirely prevent the pas-

* $\frac{n v^2 s^2}{2998.5}$ expresses the height in French inches, the quantity v being given in inches.

$\frac{n v^2 s^2}{2998.5 \times 11.25936}$ is the height in English feet, the quantity v being given in inches, for 1 English foot $= 11.25936$ French inches.

$\frac{n v^2 s^2 (11.25936)^2}{2998.5 \times 11.25936}$ expresses the height in feet, the quantity v being given in English feet.

sage of the water, and the pipe might burst before it could be forced out by a pressure at the extremity. We shall see in another chapter the means employed to obviate this difficulty and to give an issue to the confined air.

If the water be impregnated with lime or any other salt, or if it be not entirely free from muddy particles held in solution, saline incrustations or deposits of muddy matter may take place in the pipes, and materially diminish the discharge. Small seeds may occasionally find their way there, and taking root, may end by entirely stopping the passage.

Unless the effect of these various obstructions be either prevented or taken into account in determining the diameter of the pipes, we must expect to find a discharge inferior to that given by the formulæ.

49. Before leaving the subject of the theory of the motion in pipes, it seems proper to say a few words concerning some very interesting experiments made by M. Girard, on the motion of water in very small tubes, or in the case of what he calls *linear* motion; being that in which the fluid may be considered as a small cylinder, moving in the direction of its length. It results from these, that when the tube is of a certain length, which must be greater as the head and the diameter increase, the term of the formula containing the

second power of the velocity disappears, and the formula becomes $a' v = \frac{Dj}{4}$, a' being a constant coefficient to be determined by experiment; and that, when the motion has become linear, the variations in temperature exert so great an influence on the discharge, that in the interval from 0 to 86 degrees of the centigrade thermometer, the discharge varies in the ratio of 1 to 4, being greatest at the highest temperature. But as no authentic experiment has ever shown that, with the same head, the same conduit-pipe can furnish a greater quantity of water in summer than in winter, it follows that the influence of temperature on the velocity is insensible, when the motion takes place in conduit-pipes of considerable length, whose diameters are without the limits of capillarity. For a full account of these experiments, see *Mémoires de l'Institut*, 1813 – 1816.

CHAPTER III.

GENERAL REMARKS ON THE MEANS OF SUPPLYING CITIES WITH WATER.

50. When a large city is to be supplied with water, the first step to be taken is, to make a general survey of the neighbouring country, in order to ascertain whence a sufficient *quantity* can be obtained. It is of the utmost importance that the source should be so abundant as to furnish the requisite quantity even in seasons of drought; for the evils of a temporary interruption, after the inhabitants have been led to depend upon a regular supply, are so great that they must at any expense be avoided. According to the localities, recourse may be had to a stream, if there be any in the neighbourhood, or to ponds of limpid water, which may be increased in abundance by turning the neighbouring brooks into them; or artificial ponds may be formed by damming the outlet of a valley, and collecting in it the supplies from the various small streams which convey the waters which fall upon the higher grounds.

51. The quantity of rain, upon which of course depends the greater or less abundance of the streams, may be an element of some importance

to determine. It varies much in different localities.

For the following data we are chiefly indebted to the *Memoirs of the American Academy*, and to *Silliman's Journal*. The depth of rain for each year is in inches and hundredths, and includes also the water which fell in the shape of hail or snow.

BOSTON, MASS. Mean 36.45.

Years.	1818	1819	1820	1821	1822	1823
Rain.	41.59	34.62	42.68	36.82	27.16	42.30
Years.	1824	1825.				
Rain.	35.10	31.30.				

CHARLESTOWN, MASS. Mean 36.29.

Years.	1792	1793	1794	1795	1796	1797
Rain.	30.49	32.34	31.11	45.64	25.77	41.18
Years.	1798	1799	1800	1801	1802	
Rain.	38.31	33.49	45.43	40.60	34.80.	

STOW, MASS. Mean 38.28.

Years.	1792	1793	1794	1795	1796	1797
Rain.	33.91	33.59	34.98	48.38	34.47	37.53
Years.	1798	1799	1800	1801	1802	1803
Rain.	36.50	37.28	40.90	38.86	41.94	38.22
Years.	1804					
Rain.	41.05.					

WILLIAMSTOWN, MASS. Mean 32.93.

Years.	1816	1817	1818	1819
Rain.	25.98	38.36	35.21	32.16.

NEWFANE, VT. Mean 56.86.

Years.	1826	1828	1829	1830	1831
Rain.	58.20	73.30	36.10	58.50	58.60.

MARIETTA, OHIO. Mean 43.85.

Years.	1818	1819	1820	1821	1822	1828
Rain.	51.00	36.33	40.00	43.00	43.50	49.50
Years.	1829	1830	1831	1832	1833	
Rain.	39.52	37.26	53.54	48.33	40.37.	

WEST CHESTER, PENN. Mean 47.25.

Years.	1818	1821	1822	1823	1824	1825
Rain.	48.83	52.60	39.30	51.40	54.10	42.30
Years.	1826	1827				
Rain.	40.05	49.45.				

CHARLESTON, S. C. Mean 48.20.

Mean quantity of rain for 10 years between 1750 and 1786, 42.03 inches.

Ys.	1795	1796	1797	1798	1799	1800	1801
R.	71.80	58.10	55.00	45.20	75.40	51.60	42.90.

NATCHEZ, MISS. Mean 46.99.

Years.	1801	1802	1803
Rain.	45.50	57.92	37.56.

GROVE PLANTATION NEAR NATCHEZ, MISS. Mean 47.09.

Years.	1806	1807	1808
Rain.	41.90	36.89	62.48.

The topography of the country will show to a certain extent what supplies the ponds will be likely to derive from rain.

In order to ascertain the quantity which may be drawn from a river, recourse must be had to various methods of gauging its discharge per second, which will be shown in the following chapter.

52. The quantity of water necessary for a certain population does not admit of being determined with mathematical precision; since independently of what is required for domestic use, an additional quantity may be wanted in various mechanic arts. In France, one pouce or 677.91 cubic feet per day has generally been considered a supply for 1,000 persons; giving about two thirds of a foot to a person per day. In Paris, the quantity now used per day consists of 2,711,652 cubic feet furnished by the Canal de l'Ourcq for the public fountains, and for cleansing the streets, and about 200 pouces or 135,583 cubic feet of Seine water, used for cooking and drinking.* The great price of the latter water chiefly restricts its use to these purposes; and in most

* M. Genieys computes that this daily supply of 135,583 cubic feet costs the consumers 4,265,766 francs a year; being 11,714 francs or $ 2,200 per day.

houses there are wells, though the water obtained from them is not good. At Greenock in Scotland the supply was estimated at 2 cubic feet per day for each person.*

In a Report made by Messrs. P. M. Roget, William Thomas Brande, and Thomas Telford in 1828, on the supply of water to the city of London and its suburbs, we find that

"The New River Company supply daily about 2,000,000 cubic feet to between 66,000 and 67,000 tenants.

"East London Water Works supply daily 950,000 cubic feet to 42,000 tenants.

"West Middlesex Water Works supply daily 360,000 cubic feet to 15,000 tenants.

"Chelsea Water Works supply 282,000 cubic feet to 12,400 tenants.

"Grand Junction Company supply 450,000 cubic feet daily to 7,700 tenants.

"Taking the average of the whole supply, the daily consumption of each house is 23.14 cubic feet or about 180 gallons.

"The abundance is such as not only to supply all necessary demands upon ordinary and extraordinary occasions, but a proportion is constantly

* An imperial gallon contains 277.274 cubic inches, and therefore a cubic foot is equal to 6.232 gallons.

suffered to run to waste, by which the cleansing of the drains of houses, and of the common sewers is effectually accomplished, all accumulations of filth obviated, and the general healthiness of the metropolis promoted.

"The Lambeth Water Works supply 200,000 cubic feet daily to 16,000 tenants.

"The Vauxhall Water Works supply 160,000 cubic feet daily to 10,000 tenants.

"The Southwark Water Works supply 115,000 cubic feet to 7,000 tenants.

"The average of the daily supply to each house is 14.40 cubic feet or about 90 gallons.

"The quantity is sufficient except in some cases of fire, when the inconvenience appears to have arisen from the want of proper reservoirs for preserving a head of water upon the mains when the engines are not working. Much time may be lost in getting up the steam. The total amount of the quantity of water required for the daily supply of the inhabitants of the metropolis, and for the use of the manufactories requiring it, appears to be about 4,650,000 cubic feet" or 4 cubic feet to a person. (Journal of Science, Vol. XXV. Year 1828.)

53. It is also indispensable to ascertain accurately the *quality* of the water. Water may be either *hard* or *soft*. The hardness generally proceeds from saline matters or from the presence

of gas. Dr. Saunders observes * that it depends chiefly upon the sulphate and carbonate of lime held in solution; and that when these earthy salts exist in the proportion of five grains to a pint, such water will be hard. The dissolving power of hard water is less than that of soft; soap does not completely dissolve in it, but appears in a sort of curdled or coagulated state; hence it is less fit for washing, bleaching, dyeing, &c. It is also less powerful in softening the texture of vegetable leaves. Soft water has no saline or other taste, is inodorous, and dissolves soap completely. It is much to be preferred for softening the texture of animal or vegetable matters and for all culinary purposes, as well as for washing. M. Thénard says that water may be considered as good for domestic uses, when it is but slightly affected by nitrate of barytes, nitrate of silver, or oxalate of ammonia; and when, upon being evaporated, it leaves but a very small residuum.

54. These two important preliminaries as to *quality* and *quantity* being satisfactorily settled, the localities will show which should be preferred, of the various methods which present themselves for distributing the supply. Whether the water be brought from a distant and elevated

* Rees's Cyclopædia.

source, or immediately raised by machinery, the most common method is, to collect it in one or more large reservoirs situated upon elevated spots, and thence to distribute it through the streets and into the premises of individuals by means of pipes, through which it flows in virtue of its hydrostatic pressure. In some instances, however, it has been forced directly into the mains which carry it through the various quarters of the city. This last method dispenses with the cost of the construction of reservoirs, and may, in some cases, be the most economical in the first instance, but it is less regular in its operation than the other. A reservoir maintains a constant head upon the pipes, and a large body of water in reserve in case of emergency. Should an extensive fire occur, the pipes from a reservoir are at any instant in a state to do service, while it would often take some time to get the steam up, or to set other machinery in motion. Experience has proved the truth of these remarks at the London Water Works. Another great advantage of reservoirs is, that the water has time to settle and deposit the impurities which it may hold in suspension.

55. When water is to be brought from a distance, we may make use of pipes, aqueducts of masonry, or canals opened in the natural ground. If the distance be very considerable, there would

seem to be an advantage in aqueducts and canals; for the resistance which is developed in pipes of great length, diminishes the discharge in a very great degree; especially as in following the natural undulations of the ground, which must be done unless much money is laid out in trenching and embanking, there will be an increase of length and a diminution of discharge, which, our formulæ have shown us, attend upon every deviation from a straight line. On the other hand, pipes have the advantage of being out of the reach of the extremes of temperature, and, where the distance is not very considerable, will generally be the cheapest.

56. A canal in the natural ground, like an aqueduct, has the advantage of bringing a very much larger supply with but little fall. But it will generally require some time to saturate the bed so that there shall be no leakage; and it is difficult to preserve the water in as pure a state, and to protect it from animals and trespassers. It may pass through saline or calcareous earths, and thus, though naturally good, may contract a disagreeable taste or odor. In most countries it is liable to be frozen in winter. Evaporation, too, is a great source of loss, which it is impossible to avoid. Numerous experiments are needed to enable us to determine the quantity which passes off in this manner from large sheets of water.

Most of those hitherto made have been on very small quantities, and it is easily conceived that these cannot give a correct idea of the effects upon surfaces of considerable extent. Smeaton says (Reports, Vol. II.) that when deep standing waters lose by exhalation to the amount of ten inches perpendicular, it must be in a long drought of three or four months' continuance. Messrs. Sédileau and Cotte found, that at Paris the mean annual evaporation amounts to 34 inches, the mean annual quantity of rain being 21 inches. Messrs. Clausade and Pin found that the waters of the Languedoc Canal and the basin of St. Féréol, lose yearly from 30 to 32 inches by evaporation, the annual quantity of rain being about 31 inches.

From a careful consideration of all the circumstances we have enumerated, according as their influence is counteracted or increased by local causes, the engineer must determine upon the most secure and economical mode of supply.

CHAPTER IV.

MEANS OF MEASURING THE FLOW OF WATER.

57. It is a well known principle of hydraulics, first given by Torricelli, that the velocity of efflux from an aperture made in the side or bottom of a vessel, is equal to the velocity which a heavy body would acquire, in falling from the surface of the liquid to the level of the aperture. Thus h being the distance from the surface to the aperture, we shall have $v = \sqrt{2gh}$, and h is called the height due to the velocity v.

Calling ω the section of the aperture, and Q the quantity which flows out in the time t, we have, for the quantity which flows out in the time dt, $\omega v\, dt$. Hence

$$dQ = \omega v\, dt; \quad dQ = \omega \sqrt{2gh}\, .\, dt.$$

If the level of the water in the vessel be constant, h does not vary, and we shall have, by integration,

$$Q = \omega \sqrt{2gh}\, .\, t.$$

If the level of the water be variable, we must first determine the value of h as a function of t. Substituting it in the equation

$$dQ = \omega \sqrt{2gh}\, .\, dt,$$

we shall find by integration the value of Q as a function of the time.

58. It was first remarked by Newton, that the real discharge always falls short of that given by theory. This results from a phenomenon which always accompanies this kind of efflux. The stream, upon first leaving the side of the vessel, has exactly the same dimensions as the aperture through which it has passed; but at a short distance it diminishes very sensibly, the vein taking a conical or pyramidal form. This phenomenon is called the contraction of the fluid vein; and its effect is the same as a diminution in the size of the aperture.

This circumstance is generally taken into account, by introducing into the formula a certain coefficient m, representing the ratio of the real to the theoretical discharge, and always less than unity. Thus the formula would be

$$Q = \omega m \sqrt{2gh},$$

Q representing the discharge in the unit of time.

59. A great many experiments have been made by different observers, to determine the value of m. The following are the principal results.

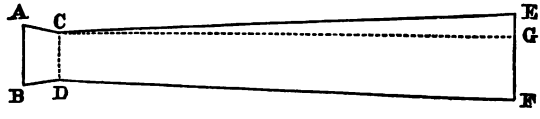
When the aperture is made in a *thin plate*, and the head is considerable, compared with its height, we have generally $m = 0.620$.

Observation has shown, that, when h diminishes, the value of m increases. Michelotti's experiments give for it the value 0.649; Venturi calls it 0.640; Bossut 0.660 and 0.667; Eytelwein 0.640; Borda 0.646. Navier says, that, when h is more than 200 times the diameter of the orifice, we have $m = 0.615$. If h diminishes so as to be successively equal to 10, 6, 4, 1 times this diameter, m will take the successive values 0.620, 0.625, 0.630, 0.642.

If a short cylindrical tube be adapted to the aperture, having a length equal to about two and a half times its diameter, the value of m will become 0.82.

With a conical tube, having as nearly as possible the form of the contracted vein, the value of m may become 0.90.

The annexed figure shows the proportions of the additional tube which, according to Venturi's



experiments, gives the greatest discharge; in which $AB = 18$ lines; $AC = 11$; $CD = 15.5$; $CG = 148$; $EF = 27$ lines.

60. The following table gives the heights *due* to various velocities, or the heights through which heavy bodies must fall to acquire various

TABLE — *continued.*

v	*h*	*v*	*h*	*v*	*h*	*v*	*h*
7.65	0.90925	8.25	1.05747	8.85	1.21688	9.45	1.38747
7.70	0.92118	8.30	1.07033	8.90	1.23067	9.50	1.40209
7.75	0.93318	8.35	1.08326	8.95	1.24453	9.55	1.41699
7.80	0.94526	8.40	1.09627	9.00	1.25848	9.60	1.43187
7.85	0.95741	8.45	1.10936	9.05	1.27250	9.65	1.44682
7.90	0.96965	8.50	1.12253	9.10	1.28660	9.70	1.46185
7.95	0.98196	8.55	1.13578	9.15	1.30078	9.75	1.47696
8.00	0.99435	8.60	1.14910	9.20	1.31503	9.80	1.49215
8.05	1.00682	8.65	1.16250	9.25	1.32936	9.85	1.50742
8.10	1.01937	8.70	1.17598	9.30	1.34377	9.90	1 52276
8.15	1.03199	8.75	1.18953	9.35	1.35826	9.95	1.53818
8.20	1.04469	8.80	1.20317	9.40	1.37283	10.00	1.55367

61. If it be required to measure the quantity of water furnished by a small stream, the whole quantity may be made to flow into a reservoir of known dimensions, and, the time of its filling being carefully noted, we can hence infer the discharge per second.

62. If the stream is of such size as to render this direct measurement impracticable, recourse may be had to another method.

A dam is built across the stream, and the whole of the water is made to pass through an orifice either vertical or horizontal, above which is maintained a constant head. The discharge may then be calculated by the following formulæ, given by Prony in a paper on this subject.*

* Mémoire sur le Jaugeage des Eaux Courantes, par M. de Prony.

When the orifice is horizontal, calling the English foot the linear unit, and making

Q = the discharge per second,
h = the height of the water above the horizontal orifice,
a = the area of the upper surface of the water,
ω = the area of the aperture,
$g = 32.1817$,
m = the coefficient of contraction,

we shall have

$$Q = m\,\omega\sqrt{\frac{2\,g\,h\,a^2}{a^2 - \omega^2}}.$$

The area a being that of the surface of the stream which is stagnant above the dam, and the area of the aperture being exceedingly small in comparison with it, the fraction $\frac{a^2}{a^2 - \omega^2}$ will differ very little from unity; neglecting this error, we may say then that the discharge is

$$Q = m\,\omega\,\sqrt{2\,g\,h}.$$

63. The formula to find the discharge through a rectangular vertical orifice is, calling

ω = the area of the orifice,
a = the height of the orifice,
b = its base, ... } $\omega = a\,b$

h = the height of the water above the top of the orifice,
k = the height above the centre,
h_1 = the height above the bottom,

$k = \frac{h + h_1}{2}$
$k = h + \frac{1}{2}a$
$k = h_1 - \frac{1}{2}a$

$g = 32.1817$,
m = the coefficient of contraction,
Q = the discharge per second,

$$Q = m \tfrac{2}{3} b \left(h_1^{\frac{3}{2}} - h^{\frac{3}{2}}\right)\sqrt{2g}.*$$

This equation may be put under the form

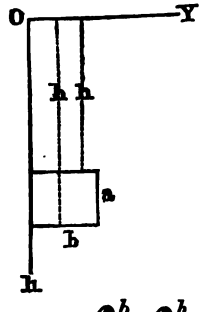
$$Q = m \tfrac{2}{3} b \left\{\left(k + \tfrac{a}{2}\right)^{\frac{3}{2}} - \left(k - \tfrac{a}{2}\right)^{\frac{3}{2}}\right\}\sqrt{2g},$$

which becomes

$$Q = m \tfrac{2}{3} bk \left\{\left(1 + \frac{a}{2k}\right)^{\frac{3}{2}} - \left(1 - \frac{a}{2k}\right)^{\frac{3}{2}}\right\}\sqrt{2gk},$$

or

$$Q = m \tfrac{2}{3} \frac{k}{a} \left\{\left(1 + \frac{a}{2k}\right)^{\frac{3}{2}} - \left(1 - \frac{a}{2k}\right)^{\frac{3}{2}}\right\} \omega \sqrt{2gk};$$

* 

The discharge or quantity which passes through any element of the aperture $a\,b$ will be

$$m\,d\,y\,d\,h\sqrt{2\,g\,h}.$$

The quantity which will pass through the whole orifice, or the sum of the elements, will be the integral of this, taken within the limits 0 and b, h and h_1;

$$Q = m \int_0^b dy \int_h^{h_1} d\,h\,h^{\frac{1}{2}}\sqrt{2g} = m\,b\,\tfrac{2}{3}\left(h_1^{\frac{3}{2}} - h^{\frac{3}{2}}\right)\sqrt{2g}.$$

and if we make

$$\tfrac{2}{3}\frac{k}{a}\left\{\left(1+\frac{a}{2k}\right)^{\frac{3}{2}}-\left(1-\frac{a}{2k}\right)^{\frac{3}{2}}\right\}=A,$$

we shall have for the discharge per second

$$Q = m A \omega \sqrt{2gk}.$$

The value of A for various values of $\frac{a}{2k}$ may be taken from the following table given by Prony, and shows the error to which we are liable, when we call the head the distance from the centre of the orifice to the surface.*

* The following is the mode of calculating the value of the factor A in the formula

$$Q = m \omega A \sqrt{2gk}.$$

We begin by developing the binomial of the formula

$$\tfrac{2}{3}\frac{k}{a}\left\{\left(1+\frac{a}{2k}\right)^{\frac{3}{2}}-\left(1-\frac{a}{2k}\right)^{\frac{3}{2}}\right\}=A.$$

$$\left(1+\frac{a}{2k}\right)^{\frac{3}{2}} = 1+\tfrac{3}{2}\frac{a}{2k}+\frac{\frac{3}{2}(\frac{3}{2}-1)}{2}\left(\frac{a}{2k}\right)^2+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)}{2\cdot 3}\left(\frac{a}{2k}\right)^3+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)}{2\cdot 3\cdot 4}\left(\frac{a}{2k}\right)^4+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)}{2\cdot 3\cdot 4\cdot 5}\left(\frac{a}{2k}\right)^5+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)(\frac{3}{2}-5)}{2\cdot 3\cdot 4\cdot 5\cdot 6}\left(\frac{a}{2k}\right)^6+\ldots \text{ \&c.}$$

$\frac{a}{2k}$	A	Log of A
0	1	0
0.1	0.99958	0.9998176 — 1
0.2	0.99832	0.9992654 — 1
0.3	0.99619	0.9983422 — 1
0.4	0.99312	0.9969974 — 1
0.5	0.98904	0.9952139 — 1
0.6	0.98383	0.9929201 — 1
0.7	0.97724	0.9900012 — 1
0.8	0.96896	0.9863058 — 1
0.9	0.95828	0.9814924 — 1
1	0.94281	0.9744242 — 1

64. In order to make sure that these formulæ will give the true result, the following precautions should be taken.

$$(1-\frac{a}{2k})^{\frac{3}{2}} = 1-\tfrac{3}{2}\,\frac{a}{2k}+\frac{\frac{3}{2}(\frac{3}{2}-1)}{2}\left(\frac{a}{2k}\right)^2-\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)}{2\cdot 3}\left(\frac{a}{2k}\right)^3+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)}{2\cdot 3\cdot 4}\left(\frac{a}{2k}\right)^4-\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)}{2\cdot 3\cdot 4\cdot 5}\left(\frac{a}{2k}\right)^5+\frac{\frac{3}{2}(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)(\frac{3}{2}-5)}{2\cdot 3\cdot 4\cdot 5\cdot 6}\left(\frac{a}{2k}\right)^6 - + \ldots \text{ \&c.}$$

$$(1+\frac{a}{2k})^{\frac{3}{2}}-(1-\frac{a}{2k})^{\frac{3}{2}} = 3\frac{a}{2k}+3\frac{(\frac{3}{2}-1)(\frac{3}{2}-2)}{2\cdot 3}\left(\frac{a}{2k}\right)^3+3\frac{(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)}{2\cdot 3\cdot 4\cdot 5}\left(\frac{a}{2k}\right)^5+3\frac{(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)(\frac{3}{2}-5)(\frac{3}{2}-6)}{2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7}\left(\frac{a}{2k}\right)^7$$

(1.) We must have above the orifice a suffi-

$$+3\frac{(\frac{3}{2}-1)(\frac{3}{2}-2)(\frac{3}{2}-3)(\frac{3}{2}-4)(\frac{3}{2}-5)(\frac{3}{2}-6)(\frac{3}{2}-7)(\frac{3}{2}-8)}{2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}\left(\frac{a}{2k}\right)^9+\text{ &c.}$$

$$(1+\frac{a}{2k})^{\frac{3}{2}}-(1-\frac{a}{2k})^{\frac{3}{2}}=\frac{3a}{2k}-\frac{1}{4\cdot2\cdot3}\frac{3a}{2k}\left(\frac{a}{2k}\right)^2-\frac{15}{(4)^2.2.3.4.5}\frac{3a}{2k}\left(\frac{a}{2k}\right)^4-\frac{945}{(4)^3.2.3.4.5.6.7}\frac{3a}{2k}\left(\frac{a}{2k}\right)^6-\frac{135135}{(4)^4.2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}\frac{3a}{2k}\left(\frac{a}{2k}\right)^8-\text{ &c.}$$

$$A=1-\frac{1}{4\cdot2\cdot3}\left(\frac{a}{2k}\right)^2-\frac{15}{(4)^2\cdot2\cdot3\cdot4\cdot5}\left(\frac{a}{2k}\right)^4-\frac{945}{(4)^3\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}\left(\frac{a}{2k}\right)^6-\frac{135135}{(4)^4\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}\left(\frac{a}{2k}\right)^8-\text{ &c.}$$

$$A=1-\frac{1}{4\cdot2\cdot3}\left(\frac{a}{2k}\right)^2-\frac{1}{(4)^2\cdot2\cdot4}\left(\frac{a}{2k}\right)^4-\frac{9}{(4)^3\cdot2\cdot4\cdot6}\left(\frac{a}{2k}\right)^6-\frac{11\cdot13}{(4)^4\cdot2\cdot4\cdot6\cdot8}\left(\frac{a}{2k}\right)^8-\frac{13\cdot15\cdot17}{(4)^5\cdot2\cdot4\cdot6\cdot8\cdot10}\left(\frac{a}{2k}\right)^{10}-\frac{15\cdot17\cdot19\cdot21}{(4)^6\cdot2\cdot4\cdot6\cdot8\cdot10\cdot12}\left(\frac{a}{2k}\right)^{12}-\text{}$$

$$A=1-\frac{1}{24}\left(\frac{a}{2k}\right)^2-\frac{1}{128}\left(\frac{a}{2k}\right)^4-\frac{3}{1024}\left(\frac{a}{2k}\right)^6-\frac{143}{98304}\left(\frac{a}{2k}\right)^8-\frac{221}{262144}\left(\frac{a}{2k}\right)^{10}-\frac{29393}{4194304}\left(\frac{a}{2k}\right)^{12}-\text{ . . . &c.}$$

cient head to prevent the convergence of the fluid lines towards the point of discharge, and to prevent the variations in the velocity of the different parts from being sensible beyond a slight distance from the orifice.

(2.) We must have a body of water sensibly calm or stagnant above the dam.

(3.) The water must be allowed to flow off with perfect freedom below the dam, so that its pressure may not obstruct the velocity of efflux or interfere with the contraction.

If we now substitute in this formula various values for $\frac{a}{2k}$, we shall have the corresponding values of A.

Suppose $\frac{a}{2k} = 0$. We have $A = 1$.

Suppose $\frac{a}{2k} = 0.1$. The first two terms will give us $A = 0.99958$, and the succeeding terms will not affect these five decimals.

In like manner

$\frac{a}{2k} = 0.2$ gives $A = 0.99832$ from 3 terms,

$\frac{a}{2k} = 0.3$ gives $A = 0.99619$ from 3 terms,

$\frac{a}{2k} = 0.4$ gives $A = 0.99312$ from 4 terms,

and so on.

(4.) The opening must be so regulated that the level of the water above it may remain constantly the same, in order that the discharge by the orifice may be exactly the measure of the quantity furnished by the stream.

(5.) A value must be given to *m*, corresponding to the circumstances of the case.

65. On account of the difficulty of complying with all these conditions, M. de Prony has given another method for finding directly the real discharge, without the necessity of guarding against so many chances of error.

Choose a part of the bed of the stream five or six hundred feet in length, at a place where a number of transverse sections may be easily measured. Erect at the lower end of this space a dam, with an opening through which the water may flow. At the upper end erect a sluice, in such a manner that it may be closed instantaneously, and thus cut off all communication with the upper water. The sluice-gate being raised to a certain height, wait till the water above it has risen to a constant level. This may be ascertained by means of a bent tube containing a float and communicating at one end with the stream. When the float remains perfectly stationary the surface of the water has become constant. As soon as this is done, close the sluice suddenly, so that the water below it may run off

through the opening in the lower dam. Note then at a number of successive instants how much the water has fallen in the basin. We suppose that a sufficient number of cross sections have been taken, so that we can find what quantity has flowed out at these successive instants. The discharge of the stream per second may now be calculated in the following manner.

Suppose

$$0, t, 2t, 3t, 4t, \ldots . nt$$

the time counted in seconds, beginning at the moment of shutting the sluice-gate;

$$0, q_{\mathrm{I}}, q_{\mathrm{II}}, q_{\mathrm{III}}, q_{\mathrm{IV}}, \ldots . q_{n}$$

the volume of water which had passed out at the corresponding instants.

The volume Q, which the stream supplies per second, will be found by one of the following equations.

If only one observation be made,

$$Q = \frac{1}{t}\, q_{\mathrm{I}}.$$

If two observations,

$$Q = \frac{1}{t}\left(2q_{\mathrm{I}} - \frac{q_{\mathrm{II}}}{2}\right).$$

If three observations,

$$Q = \frac{1}{t}\left(3q_{\mathrm{I}} - 3\,\frac{q_{\mathrm{II}}}{2} + \frac{q_{\mathrm{III}}}{3}\right).$$

If n observations

$$Q=\frac{1}{t}\left(nq_1-\frac{n(n-1)}{1\cdot 2}\frac{q_{11}}{2}+\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\frac{q_{111}}{3}-\ldots\pm\frac{q_n}{n}\right).$$

The quantities q_1, q_{11}, q_{111}, ... q_n are respectively multiplied by the coefficients of the binomial formula, and then divided by the series of natural numbers 1, 2, 3 ... n. The method consists thus in finding from a series of observations, by means of a formula of interpolation, the law according to which the water flows; and thence determining the value of the discharge at the first instant after the sluice is closed, and the time begins to be counted.

66. When we have to measure the quantity of water flowing in large rivers, where none of the preceding methods can be applied, we must measure a section of the river, and determine the mean velocity of the stream as it passes this section. The product of these two quantities will give the discharge.

67. The most common method of finding this mean velocity is, to measure the velocity at the surface in the middle of the stream, and to deduce from that observation the value of the mean. Dubuat made a number of experiments to determine the relation between these two velocities, and succeeded in representing, with considerable exactness, the result of his experiments by the formula

$$v = (\sqrt{V} - 0.5)^2 + 0.25,$$

in which v is the mean velocity, V that at the surface, and the French inch the unit.

M. de Prony also attended to this subject and gave the formula

$$v = \frac{V\,(V + 2.37187)}{V + 3.15312},$$

the metre being the unit.

This formula is more exact than that of Dubuat, and seems more rational also. The first, for instance, gives a mean velocity of 0.25, when the velocity at the surface V is nothing.

Changing the constant quantities in M. de Prony's formula, so as to make it answer for English measures, we shall have *

$$v = \frac{V\,(V + 7.78188)}{V + 10.34508},$$

when the English foot is the unit.

68. From this formula the following table has been calculated.

* Suppose V given in English feet; we might reduce it to metres by multiplying it by 0.30479 and enter it in the above formula, which thus becomes

$$\frac{0.30479\,V(0.30479\,V + 2.37187)}{0.30479\,V + 3.15312}.$$

This value of v is now in metres. In order to reduce it to feet, we divide it by 0.30479, and we have

$$v = \frac{V(0.30479\,V + 2.37187)}{0.30479\,V + 3.15312} = \frac{V(V + 7.78188)}{V + 10.34508}$$

for the English formula, v and V being both in feet.

TABLE.

Velocity. Surface.	Velocity. Mean.	Differences.	Velocity. Surface.	Velocity. Mean.	Differences.	Velocity. Surface.	Velocity. Mean.	Differences.
feet.	feet.		feet.	feet.		feet.	feet.	
0.05	0.038		2.15	1.709		4.25	3.504	
		37			41			43
0.10	0.075		2.20	1.750		4.30	3.547	
		38			42			44
0.15	0.113		2.25	1.792		4.35	3.591	
		38			42			44
0.20	0.151		2.30	1.834		4.40	3.635	
		39			42			44
0.25	0.190		2.35	1.876		4.45	3.679	
		38			41			44
0.30	0.228		2.40	1.917		4.50	3.723	
		38			42			44
0.35	0.266		2.45	1.959		4.55	3.767	
		39			42			44
0.40	0.305		2.50	2.001		4.60	3.811	
		38			42			44
0.45	0.343		2.55	2.043		4.65	3.855	
		39			42			44
0.50	0.382		2.60	2.085		4.70	3.899	
		39			42			44
0.55	0.421		2.65	2.127		4.75	3.943	
		38			42			45
0.60	0.459		2.70	2.169		4 80	3.988	
		39			43			44
0.65	0.498		2.75	2.112		4.85	4.032	
		40			42			44
0.70	0.538		2 80	2.254		4.90	4.076	
		39			42			44
0.75	0.577		2.85	2.296		4.95	4.120	
		39			43			45
0.80	0.616		2.90	2.339		5.00	4.165	
		39			42			44
0·85	0.655		2.95	2.381		5.05	4.209	
		40			43			45
0.90	0.695		3.00	2.424		5.10	4.254	
		39			42			44
0.95	0.734		3.05	2.466		5.15	4.298	
		40			43			45
1.00	0.774		3 10	2.509		5.20	4.343	
		40			43			44
1.05	0.814		3.15	2.552		5.25	4.387	
		40			42			45
1.10	0.854		3.20	2.594		5.30	4.432	
		40			43			44
1.15	0.894		3 25	2.637		5.35	4.476	
		40			43			45
1.20	0.934		3.30	2.680		5.40	4.521	
		40			43			45
1.25	0.974		3.35	2.723		5.45	4.566	
		40			43			44
1.30	1.014		3.40	2.766		5.50	4.610	
		40			43			45
1.35	1.054		3.45	2.809		5.55	4.655	
		40			43			45
1.40	1.094		3.50	2.852		5 60	4.700	
		40			43			45
1.45	1.135		3.55	2.895		5.65	4.745	
		40			43			44
1.50	1.175		3.60	2.938		5.70	4.789	
		40			44			45
1.55	1.216		3.65	2.982		5.75	4.834	
		41			43			45
1.60	1.257		3.70	3 025		5.80	4.879	
		40			43			45
1.65	1.297		3.75	3.068		5.85	4 924	
		41			43			45
1.70	1.338		3.80	3.111		5.90	4.969	
		41			44			45
1.75	1.379		3.85	3.155		5.95	5.014	
		41			43			45
1.80	1.420		3.90	3.198		6.00	5.059	
		41			44			45
1.85	1.461		3.95	3.242		6.05	5.104	
		41			43			45
1.90	1.502		4.00	3.285		6.10	5.149	
		41			44			45
1.95	1.543		4.05	3.329		6.15	5.194	
		42			43			46
2.00	1.585		4.10	3.372		6.20	5.240	
		41			44			45
2.05	1.626		4.15	3.416		6 25	5.285	
		41			44			45
2.10	1.667		4.20	3.460		6.30	5.330	
		42			44			45

TABLE — *continued.*

Velocity. Surface.	Velocity. Mean.	Differences.	Velocity. Surface.	Velocity. Mean.	Differences.	Velocity. Surface.	Velocity. Mean.	Differences.
feet.	feet.		feet.	feet.		feet.	feet.	
6.35	5.375	45	7.60	6.514	46	8.85	7.668	47
6.40	5.420	46	7.65	6.560	46	8.90	7.715	46
6.45	5.466	45	7.70	6.606	46	8.95	7.761	47
6.50	5.511	45	7.75	6.652	46	9.00	7.808	46
6.55	5.556	46	7.80	6.698	46	9.05	7.854	46
6.60	5.602	45	7.85	6.744	46	9.10	7.900	47
6.65	5.647	45	7.90	6.790	46	9.15	7.947	47
6.70	5.692	46	7.95	6.836	46	9.20	7.994	46
6.75	5.738	46	8.00	6.882	46	9.25	8.040	47
6.80	5.784	45	8.05	6.928	46	9.30	8.087	46
6.85	5.829	45	8.10	6.974	47	9.35	8.133	47
6.90	5.874	46	8.15	7.021	46	9.40	8.180	46
6.95	5.920	46	8.20	7.067	46	9.45	8.226	47
7.00	5.966	45	8.25	7.113	46	9.50	8.273	47
7.05	6.011	46	8 30	7.159	46	9.55	8.320	46
7.10	6.057	46	8.35	7.205	46	9.60	8.366	47
7.15	6.103	45	8.40	7.251	47	9.65	8.413	47
7.20	6.148	46	8.45	7.298	46	9.70	8.460	46
7.25	6.194	46	8.50	7.344	46	9.75	8 506	47
7.30	6.240	45	8.55	7.390	47	9.80	8.553	47
7.35	6.285	46	8.60	7.437	46	9.85	8.600	47
7.40	6.331	46	8.65	7.483	46	9.90	8.647	46
7.45	6.377	46	8.70	7.529	46	9.95	8.693	47
7.50	6.423	46	8.75	7.575	47	10.00	8.740	
7.55	6.469	45	8.80	7.622	46			

M. de Prony has also given the very simple formula

$$v = 0.816458\ V,$$

which, simple as it is, is yet more correct than that of Dubuat.

69. There are several ways of measuring the velocity at the surface. The first we shall mention is Pitot's tube. (Plate I, Fig. 1.) This is a tube say two inches in diameter, and five or

six feet long, one end of which is bent so as to make a right angle, and terminates in a cone, at the summit or end of which is a small opening. When this is placed in the water with the small opening opposed to the stream, the force of the current pressing against it, forces the water to rise in the tube to a certain height, which, on account of the velocity of the stream, will be greater than the height of the surrounding water. According to Dubuat's experiments $\frac{1}{8}$ of the excess of height will be the height to which the velocity is due; and this being found, the velocity is deduced from it at once.

The height to which the water rises in the tube is marked by the stem of a small float, which is placed in the upper end of the tube, and moves against a graduated scale. It is evident that this height will be greatest when the tube is directly opposed to the current, therefore it should be turned in various directions till the highest mark is found. Then turning it in the opposite direction, and noting the lowest position of the stem, the difference between this and the former mark will be the difference sought.

If a long, pointed stick, having near its end a broad plate, be driven into the bed of the river, this plate preventing it from entering the ground any farther during the time of the experiment, the tube may be held fast or tied to this stick,

which may have a scale marked upon it; and this will be the easiest way of making the experiment.

In this manner the velocity in various points of the same vertical line may be determined, and that in different verticals, and from the mean of them we can find the mean velocity; knowing also, by sounding, the depths of the water in various parts of the same section, and measuring the width, we know the area of the section, and consequently the discharge.

70. The velocity at the surface may be determined also by means of little balls of wax, or of some substance of nearly the same specific gravity as water, which are made to float near the surface, and are carried along by the current. They should be of such weight as to be just immersed in the water, in order that the wind may not affect them. This experiment should be often repeated at different times, and in calm weather, and a mean of the results taken. A mean of the velocities at different heights, may be obtained by loading a stick in such a manner that it will stand upright in stagnant water. When placed in the stream, it will move in virtue of the mean velocity of the different parts in contact with it, and its inclination backward or forward will show whether the velocity increases or diminishes towards the bottom.

71. A spherical body suspended by a fine wire or thread may be presented to the current, and the velocity deduced from the inclination of the thread.

72. The pressure which the water exerts against a disk of known dimensions, may be measured by means of a dynamometer. We can thence determine the velocity, since we may say in practice without any sensible error that the effort of water against a fixed vertical surface, is equal to the weight of a prism whose base is this surface and whose height is the height due to the velocity.

73. It is sometimes necessary to measure the quantity of water which flows over a dam. Let l be the width of the stream, h the height of the surface above the edge of the dam, which should be measured at a little distance back in order to avoid the depression which takes place just over the dam. If the banks or sides slant in such a manner as to lead the water to the overfall with as little obstruction as possible, calling Q the discharge per second, we have as the result of Dubuat's experiments $4.5756\, l \sqrt{h^3} = Q$, the English foot being the unit. But if, as is generally the case, the sides or banks do not slope towards the dam, the *contraction* takes place, and the quantity which flows over will only be $\frac{3}{4}$ of the above. In that case the formula becomes

$$Q = 3.4317\ l\ \sqrt{h^3}.$$

According to Bidone's experiments the formula would be

$$Q = 3.2242\ l\ \sqrt{h^3}.$$

According to Dr. Robison, we have in a similar case, for small apertures, less than 18 inches wide,

$$Q = 3.3515\ l\ \sqrt{h^3};$$

for larger apertures

$$Q = 3.5583\ l\ \sqrt{h^3}.$$

74. In general, M. Girard observes, when we measure the quantity of water furnished by a stream, it is in order to find the supply which it can furnish for our use in various mechanical operations. The question then is, not so much what quantity passes at a certain instant, as what is the mean product for a length of time. We ought, then, strictly speaking, to measure the discharge which is continually varying, every day in the year, in order to find a true mean. As we cannot do this, we ought not to omit taking a measure at the time of lowest water, since it is very important to ascertain the minimum supply. If there are mills upon the stream, it is very difficult to get an exact measure, for according as the water is held back or let out, the quantity will vary considerably.

75. A great number of experiments have been made in Italy by various observers, to determine

the *scale of velocities*, or the comparative velocities at various points of the same vertical line.

From these experiments it may be inferred, that if we represent the velocities at various points by the ordinates of a curve, it will have such a figure as No 1. (Plate I. figure 2.) From a report made by Messrs. de Prony, Girard, and Navier to the Academy of Sciences of Paris, upon some experiments made by M. Raucourt on the velocity of the Neva, we infer that the results of his experiments would give in winter when the river was covered with ice, a curve resembling No. 2. In summer, the velocities would be represented as in No. 3, by the curve AB in calm weather, AC with a wind against the stream, and AD with a wind in the direction of the stream. Of course, the violence of the wind and its duration have an influence upon the effect which it produces.

According to Dubuat's experiments, the curve would be No. 4. From M. Raucourt's experiments it would seem that the state of the atmosphere should be taken into consideration, if we seek to determine the mean velocity from that at the surface.

CHAPTER V.

CONVEYANCE OF WATER BY CANALS OR AQUEDUCTS.

76. The formulæ of Chapter I. show us the relation which exists between the section of the bed, the quantity which flows per second, the velocity, inclination, and depth; so that we have nothing to do, but to substitute in these formulæ the numerical values which other considerations have led us to affix to some of these elements, and thence deduce the appropriate values of the rest.

77. Generally speaking, the section of a canal will have the trapezoidal form, since a certain inclination is necessary to prevent the banks from falling in. In this case the mean radius may be represented by

$$R = \frac{\omega}{\chi} = \frac{h\,(w + n\,h)}{w + 2\,h\,\sqrt{1 + n^2}}.$$

In which

ω = area of the section,
χ = perimeter,
h = depth of water,
w = width at bottom,
n = ratio of base of slope to its height.

In case of a rectangle we have $n = 0$, and the value of R becomes

$$R = \frac{h\,w}{w + 2\,h}.$$

If the width is very great compared with the depth, as is the case in broad, shallow rivers, $2h$ may be omitted in the denominator, and the mean radius will be very nearly the depth h. The value of n evidently depends upon the nature of the soil, and ranges generally between 1 and 2.

78. There are various causes which render it necessary to increase considerably the dimensions of an open canal, which is intended to convey a supply of water. The evaporation from its exposed surface is a cause of loss from which there are no means of escaping. The quantity which will pass off in this manner varies greatly in different localities, and can only be accurately known by a long series of careful observations. (See Chapter III.)

79. Leakage is also a danger to be guarded against, by augmenting the supply in the first place, and also by various other means. The loss due to this cause depends upon the nature of the soil. It is generally much greater when the water is first introduced into the canal, than when it has had time to saturate the bed through which it flows. In a rocky and loose soil, this

saturation may not take place, and recourse must be had to artificial methods for retaining the water.

80. As instances of the great amount of leakage in certain soils, and of a proper method of preventing it, we may mention first the Canal du Centre, in France. This canal is carried along the side of a hill in the neighbourhood of the quarries of Vertempierre; and its sides were formed by a dry wall, behind which had been rammed a lining of clay 3.28 feet in thickness. The leakage through this and through the embankment was such, however, that in the space of 800 feet in length, there was found to be a daily loss of from four to five hundred thousand cubic feet. The method which effectually remedied the evil was, to line the sides and the bottom, throughout this whole length, with rubble masonry, of which one foot in depth was laid in hydraulic mortar. This mortar was employed in large quantities, and great care was taken to fill the joints perfectly. By this means an effectual stop was put to the leakage.

The Canal of St. Quentin, in the north of France which was finished in 1811, lost at first such a great quantity of water, that the boats were delayed two or three months in going from Cambrai to Chauny, which passage they now make in twelve days. When the water was first

introduced into the Canal de Narbonne, in 1787, the leakage was enormous, and ten months afterwards, when it was considered much less than at first, the canal lost in six days water enough to fill it entirely. In 1800 it took 21 days for the same quantity to pass off in this manner. The feeder of St. Privé, which supplies the canal de Briare, though it has been in use nearly 200 years, still loses three quarters of its water. The feeders of the Languedoc canal lose in twelve days almost enough to fill them. A method often employed to put a stop to this evil is, to introduce muddy water into the canals, in order that the mud may be deposited and enter the interstices of the permeable bed. But MM. Fèvre and Robillard, in stopping the leaks of a part of the Canal de Bourgogne, found that it was a much more expeditious and effectual method to employ fine sand. This sand is thrown with the shovel in small quantities at a time, in such a manner as to scatter it over the surface of the water; and thus falling in a very divided state, the grains are drawn into the little interstices of the bed which they soon fill. This process was used in a great number of places with complete success, and a few hours' time was sufficient to produce a most advantageous effect.*

* Annales des Ponts et Chaussées. 1832.

81. When we have fixed upon the proper quantity of water for the canal to convey, we must determine also what velocity we will give it. In some instances, as in that of the Canal de l'Ourcq, which conveys water for the public fountains in Paris, the canal is intended to serve the purposes of navigation, as well as to fulfil its primary object. In such a case it is proper to give as small a velocity as possible to the water, in order to oppose the least possible resistance to boats when towed against the stream. The French engineers determined in this case that the least velocity sufficient to preserve the water pure and fresh, was 35 centimetres or 1.15 feet per second. This depends evidently in a degree upon the nature of the soil, and of the plants which grow in the bed of the canal; for all will not have an equally injurious effect upon the water which remains in contact with them.

82. The following Table shows the velocity of the water in various natural streams and aqueducts.*

* Genieys. Essai sur les Moyens de conduire, d'élever, et de distribuer les Eaux.

TABLE.

Name.	Slope.	Velocity per second.	Remarks.
		feet.	
Tiber		3.281	At Rome, when the water is low.
Seine	0.000125	2.559	According to M. de Chézy's observation between Surène and Neuilly, the height being 1.26 metre above the summer level.
Loire	0.000382	4.265	
Rhone at Arles		4.790	When the water is low.
Rhone at Baucaire		8.530	" " "
Durance		8.530	Common velocity between Sisteron and its mouth; the water being not more than 3 metres above the summer level.
Feeder of Courpalet (Canal of Orléans)	0.0000355	0.295	
Canal de l'Ourcq	0.0000625	1.230	Calculated velocity supposing a depth of 4.92 ft.
Do.	0.0001236	1.772	Do. Do.
Roman Aqueducts	0.001543 0.002315		
Aqueduct of Nismes	0.000400	2.001	Velocity calculated according to the formula.
Bridge of Gard			
Aqueduct of Mont Pyla, at Lyons	0.0016667	2.953	Do.
Aqueduct of Metz	0.0010034	2.789	Do.
Aqueduct of Trappes	0.000125	1.772	From an experiment by Picard.
Aqueduct of Caserta	0.000208	1.345	From the formula.
Aqueduct of Montpellier	0.000289	0.712	Do.
Feeder of St. Privé	0.000078		
Canal of Briare			
Feeder of the Canal du Centre	0.000278		
Aqueduct of Arcueil	0.000417		
Aqueduct of Roquencourt	0.000294		
Aqueduct of Maintenon	0.000210		

83. It should be observed, that the plants which grow in the bed of the stream diminish the velocity considerably. Dubuat, when making

being supposed to be built with sufficient care, and good mortar being employed, we may consider ourselves as secure from any loss by means of leakage, so that no enlargement of dimensions is required to counterbalance that evil. Moreover there is nothing to fear from the growth of reeds or other plants which might retard the flow; nor is the same velocity necessary to keep the water sweet and wholesome, since there are no noxious herbs or saline earths with which it may remain in contact.

87. In building these aqueducts the object generally is, to give such an easy passage to the water that a very small head will be sufficient, so that it may be brought to the most elevated parts of a city, with but little loss of height in its passage. Sometimes, however, to cross a deep valley and maintain a uniform inclination, would require an enormous expense of masonry, and the passage may there be made by other means. From one side of the valley the water from the aqueduct is received into a pipe, by means of which it is conveyed along the bottom or over a low bridge to a less elevated spot on the opposite side, where it is again received into another branch of the aqueduct. A sufficient head may be allowed to give the water considerable velocity in the pipe, which will generally not need to be very long. Thus the advantages of an aqueduct may be gained throughout

the greater part of the line, though they will be lost here for a certain distance. This method is used in the aqueduct of Genoa.

88. If we wish to convey the water of an aqueduct over a hill, provided its height be not more than 34 feet, we can use a syphon. As it would not be easy to exhaust the air from a syphon of such a size, the proper way to put it in operation would be to close the two ends, and fill it from an opening in the top; the ends being then opened, the water will flow. In order to prevent the air from afterwards entering the pipe, the extremities should be kept under water. There should also be at the summit a cock or air valve. (See Articles 120 and following.)

89. In order to exhibit as distinctly as possible the mode of applying the preceding rules and formulæ, we will take an example and go through the calculations.

We will suppose a pond situated at a distance of 10 miles and of sufficient extent to furnish an ample supply. The quantity which we wish to convey is 500,000 cubic feet per day, equivalent to 5.787 cubic feet per second. We suppose the section of the aqueduct to be a trapezoid, with sides sloping at one fifth; and that the fall from the surface of the pond to the level

of the reservoir into which the aqueduct opens is 40 feet. Thus

l . . . length $= 52800$ feet,

z . . . fall $= 40$,

I . . . inclination $= \frac{40}{52800} = 0.0007576$,

Q . . . discharge per second $= 5.787$ cubic feet,

n . . . ratio of base of slope of sides to their height $= \frac{1}{2}$.

We suppose also that the width w of the aqueduct at bottom is equal to twice the height at which the water stands. The question is, what dimensions should we give to the aqueduct, and what will be the velocity of the water.

We shall use Eytelwein's equation given in Article 28.

$$0.0000242651\ Q\omega + 0.0001114155\ Q^2 = \frac{\omega^3}{\chi} I.$$

We have, according to Article 77,

$\omega = h\ (w + nh)$,

$\chi = w + 2h\sqrt{1+n^2}$,

and, since $w = 2h$ and $n = \frac{1}{2}$,

$\omega = h\ (2h + \frac{1}{2}h) = \frac{11}{2}h^2$,

$\chi = 2h + 2h\sqrt{1+\frac{1}{4}} = 2h + 2.0396h = 4.0396h$.

Substituting these values in the equation, it becomes

$$0.0000242651 \times 5.787 \times \frac{11}{5}h^2 + 0.0001114155 \times (5.787)^2 = \frac{(11)^3\ 0.0007576\ h^5}{(5)^3\ 4.0396}.$$

Performing the multiplications and dividing by the coefficients of h^5, we obtain

$$h^5 - 0.1547\, h^2 = 1.8685.$$

It is evident by inspection, that the value of h which will satisfy this equation of the 5th degree, is a little greater than unity. After two or three trials we shall find that it is very nearly

$$h = 1.157.$$

Consequently

$$w = 2.314.$$

Moreover

$$v = \frac{Q}{\omega} = \frac{5.787 \times 5}{11 \times (1.157)^2} = 1.965 \text{ feet.}$$

The velocity at the surface V, according to the Table of Article 68, will be 2.457.

We must therefore give the aqueduct a width of 2.314 feet at the bottom. The water will stand at the height of 1.157, and will have a velocity of 2.457 feet at the surface and a mean velocity of 1.965.

90. We might now propose another question. Let us suppose that there are in the immediate neighbourhood of the city four suitable locations for a reservoir, which are at such a height that the surface of the water would stand at the respective levels of 30, 40, 50, and 60 feet below the pond. The aqueduct is to be built with sloping sides as before. We intend that the water shall stand in it at the depth of 2 feet, and

that the width at the bottom shall be 4 feet. What will be the mean velocity of the water, and what will be the quantity conveyed to either of these summits?

We have as before

$$\omega = \tfrac{11}{5} h^2 = \tfrac{44}{5} = 8.8,$$
$$\chi = 4.0396 \times h = 8.0792,$$
$$R = \frac{\omega}{\chi} = \frac{8.8}{8.0792} = 1.0892,$$
$$Q = 8.8\, v.$$

To

$z =$	30	40	50	60	correspond respectively
$I =$	0.0005682	0.0007576	0.0009470	0.0011363	
$RI =$	0.0006189	0.0008252	0.0010338	0.0012376.	

We now have recourse to the Table of Article 29 to find the mean velocities corresponding to these several values of RI, and we shall use Eytelwein's numbers.

To $RI = 0.0006189$ corresponds $v = 2.25$.

For $RI = 0.0008252$, v falls between 2.60 and 2.65, and we can find it by a simple proportion. The difference between the values of RI corresponding to those velocities is 0.0000304.

We have also

$$0.0008252 - 0.0008163 = 0.0000089;$$

and we have

$$304 : 89 :: 0.05 : 0.015.$$

Hence $v = 2.60 + 0.015 = 2.615$.

In a similar manner we find for

$RI = 0.0010338$ $\quad v = 2.939$

$RI = 0.0012378$ $\quad v = 3.226.$

We have then, in the four successive cases,

	ft.	ft.	ft.	ft.
$v =$	2.250	2.615	2.939	3.226
	c. ft.			
$Q = 8.8\,v =$	19.800	23.012	25.863	28.389
per day	1,710,720	1,988,237	2,234,580	2,452,787

91. A third question may be proposed which seems a little more complicated, but it may be solved with the same facility as the preceding.

Three reservoirs are to be supplied by means of the same aqueduct. At a certain distance from the fountain-head, it discharges 1 cubic foot per second, which is conveyed to the first reservoir by a channel 6000 feet in length, with a fall of 2 feet. At some distance farther, it discharges 2 cubic feet per second, which is conveyed to the second reservoir by a channel having a length of 10,000 feet, and a fall of 1 foot. The aqueduct is then continued till it reaches the third reservoir, into which it discharges 24 cubic feet per second. We suppose that from the fountain-head to its extremity it has a rectangular section, 4 feet in width, and that the inclination throughout its whole length is 1 foot in 1000, or that $I = 0.001$.

We wish to know what will be the depth of the water and the velocity in the three divisions

of the principal aqueduct; and also the dimensions of the two small aqueducts leading to the first two reservoirs, and the velocity of the water in them.

In the first portion of the large aqueduct we have

$$Q = 27,\quad I = 0.001,\quad \omega = 4h,\quad \chi = 4 + 2h.$$

Substituting these values in the third equation of Eytelwein (Article 28), it becomes

$$0.0000242651 \times 27 \times 4h + 0.0001114155 \times 27 \times 27 = \frac{4^3 h^3}{4 + 2h}\, 0.001;$$

whence we deduce

$$h^3 - 0.0819\, h^2 - 2.702\, h = 5.076;$$

and a few trials will show us that this equation is satisfied by giving to h the value 2.265. This, then, is the depth in the first part of the aqueduct. The velocity is

$$\frac{Q}{\omega} \text{ or } \frac{27}{4 \times 2.265} = 2.98.$$

In the second portion of the aqueduct we have

$$Q = 26,\quad I = 0.001,\quad \omega = 4h,\quad \chi = 4 + 2h.$$

The formula becomes, therefore,

$$0.0000242651 \times 26 \times 4h + 0.0001114155 \times 26 \times 26 = \frac{4^3 h^3}{4 + 2h}\, 0.001;$$

whence we deduce

$$h^3 - 0.0789\, h^2 - 2.5113\, h = 4.7073;$$

and this equation is satisfied by making $h = 2.20$. This, then, is the depth in the second portion. The velocity is

$$\frac{Q}{\omega} = \frac{26}{4 \times 2.2} = 2.95.$$

In the third portion of the aqueduct we have

$$Q = 24, \quad I = 0.001, \quad \omega = 4h, \quad \chi = 4 + 2h.$$

The formula becomes, therefore,

$$0.0000242651 \times 24 \times 4h + 0.0001114155 \times 24 \times 24 = \frac{4^3 h^3}{4 + 2h} 0.001;$$

whence we deduce

$$h^3 - 0.0728 h^2 - 2.1511 h = 4.011,$$

and this equation is satisfied by making $h = 2.06$. This, then, is the depth in the last portion. The velocity is

$$\frac{Q}{\omega} = \frac{24}{4 \times 2.06} = 2.91.$$

There remain now to be considered the two small aqueducts. We will make them both rectangular and the width of the first shall be 1 foot. We shall have, then, for the first of them,

$$Q = 1, \zeta = 2, \lambda = 6000, w = 1, \omega = h, \chi = 1 + 2h.$$

The formula becomes, therefore,

$$0.0000242651\, h + 0.0001114155 = \frac{h^3}{1 + 2h} \frac{1}{3000};$$

whence we deduce

$$h^3 - 0.1456\,h^2 - 0.7413\,h = 0.3342;$$

and this equation is satisfied by making $h = 1.10$, which will therefore be the depth of the water in this aqueduct. The velocity will be

$$\frac{Q}{\omega} = \frac{1}{1.1} = 0.91.$$

Let the width of the second aqueduct be 2 feet. We shall have

$$Q = 2,\ \zeta = 1,\ \lambda = 10000,\ w = 2,\ \omega = 2\,h,\ \chi = 2 + 2h.$$

The formula becomes, therefore,

$$0.000242651 \times 2 \times 2\,h + 0.0001114155 \times 4 = \frac{8\,h^3}{2 + 2\,h} 0.0001;$$

whence we deduce

$$h^3 - 0.2427\,h^2 - 1.3568\,h = 1.1142;$$

and this equation is satisfied by making $h = 1.56$, which will therefore be the depth of the water in this aqueduct. The velocity will be

$$\frac{Q}{\omega} = \frac{2}{2\,h} = 0.64.$$

92. The equation $a\,v + b\,v^2 = R\,I$ of Article 24 shows that when the velocity v, and the inclination I are determined, R is also determined by means of them. But since $R = \frac{\omega}{\chi}$, it is evident that the value of R thus found will correspond to a number of channels having different sec-

tions; for we may alter simultaneously both ω and χ, without altering the relation which these two quantities bear to each other, or the value of $\frac{\omega}{\chi}$.

Thus suppose, for instance, that we have $v = 5$ and $I = 0.001$. The Table of Article 29 gives us for $v = 5$ the corresponding value $RI = 0.0029067$. Whence we deduce $R = 2.9067$; and any section such that its area ω divided by its perimeter χ is equal to 2.9067, will be that of an open channel which with an inclination of 0.001 would convey water with a mean velocity of 5 feet per second.

The trapezoid is the most common section given to canals, and we have seen (Art. 77), that the general expression for the mean radius in that case is, $R = \frac{h(w + nh)}{w + 2h\sqrt{1+n^2}}$; we have then $\frac{h(w + nh)}{w + 2h\sqrt{1+n^2}} = 2.9067.$

Suppose in the first place $n = 0$, in which case the trapezoid becomes a rectangle, and we have

$$\frac{hw}{w + 2h} = 2.9067,$$

whence we deduce

$$w = \frac{2.9067}{h}\, w + 5.8134.$$

It is evident that this equation will not give a positive value for w unless the coefficient of w in the second member is a fraction, or unless h be greater than 2.9067; and that every value we may give to h greater than 2.9067 will furnish a positive value for w, and thus give the section of a channel answering the conditions with which we set out.

Let $h = 8$; $w = 0.9689\,w + 5.8134$; whence $w = 186.93$.
Let $h = 6$; $w = 0.4845\,w + 5.8134$; whence $w = 11.277$.
Let $h = 10$; $w = 0.29067\,w + 5.8134$; whence $w = 8.1956$.

Thus a channel with a rectangular section having either of the values of w for its width, and the corresponding h for its depth, would convey water with the velocity of 5 feet in a second, the inclination being 0.001.

Suppose now that $n = \frac{1}{5}$, or that the sides of the channel have a slope of $\frac{1}{5}$. We have

$$2.9067 = \frac{h\,(w + \frac{h}{5})}{w + 2h\sqrt{1 + \frac{1}{25}}} = \frac{w\,h + \frac{h^2}{5}}{w + 2.0396\,h};$$

whence we deduce

$$w = \frac{h\,(\frac{h}{5} - 5.9286)}{2.9067 - h}.$$

In order that the value of w may be positive, it is necessary that the numerator and denominator of the second member of this equation should have the same sign.

If $h > 2.9067$, the denominator is negative ;
If $h < 29.643$, the numerator is negative.
If $h < 2.9067$, the denominator is positive ;
If $h > 29.643$, the numerator is positive.

From this it is evident that the numerator and denominator cannot both be positive; and that they will both be negative when h is included between 2.9067 and 29.643.

Let $h = 3$; $w = \frac{3(0.6 - 5.9286)}{2.9067 - 3} = 171.34.$

Let $h = 6$; $w = \frac{6(1.2 - 5.9286)}{2.9067 - 6} = 9.172.$

Let $h = 10$; $w = \frac{10(2 - 5.9286)}{2.9067 - 10} = 5.5385.$

Thus if the width of the channel be either of these values of w, and the depth equal to the corresponding values of h, the sides sloping at one fifth, the mean velocity will be 5 feet per second.

By calculating the area and perimeter of these sections, we reverse the operation we have performed, and test its correctness. Thus in the rectangular section,

$h = 10$, $w = 8.1956$, $\omega = 81.956$, $\chi = 28.1956$,

$$R = \frac{81.956}{28.1956} = 2.9067 ;$$

and in the trapezoidal section,

$h = 10$, $w = 5.5385$, $\omega = 10(5.5385 + 2)$, $\chi = 5.5385 + 20.396$,

$$R = \frac{10(5.5385 + 2)}{5.5385 + 20.396} = 2.9067.$$

93. Hitherto we have always supposed a certain inclination in the bed of the stream, as will almost always be the case. But a question may arise as to what the motion would be in a horizontal channel, at one extremity of which the water has no outlet. In this case, the surface of the water will not be parallel with the bed, but there will be an inclination towards the outlet, and the conditions of the *uniform motion*, which require that the section should be the same throughout, are no longer fulfilled. The equation of Article 21 is therefore not applicable here, and we must have recourse to the general formula of Article 20. The manner of applying it was given by Bélanger, and will best be explained by an example.

94. Suppose a canal 1600 feet long, with a horizontal bed. It is 5 feet wide at the bottom, and the sides have a slope of $\frac{1}{3}\frac{1}{0}$. It receives at one extremity a constant supply of 30 cubic feet per second. This whole quantity is discharged at the other extremity. The question is, what will be the depth of the water in the canal, at the end where the supply is introduced, supposing that, at the other extremity, the water stands just 2 feet above the floor.

The inclination being nothing, we must make $i = 0$ in the formula of Article 20. Multiplying also the two terms by ω, it becomes

$$ds = \frac{\frac{v^2 x}{g} - \omega}{\chi(a v + b v^2)} d h,$$

which is to be integrated (Art. 23) between two limits, h_m which is unknown, and $h_0 = 2$.

We have from the data of the question

$$\int_{h_m}^{h_0} ds = 1600.$$

95. It may be observed that since h diminishes as s increases, in order to have ds and dh positive at the same time, the sign of the above equation should be changed, and we shall have

$$ds = \frac{\omega - \frac{v^2 x}{g}}{\chi(a v + b v^2)} d h,$$

which is to be integrated from $h_0 = 2$ to h_m; this last limit being such as to give us

$$\int_{h_m}^{h_0} ds = 1600.$$

From the data of Article 94 we have

$$x = 5 + \tfrac{2}{3\sqrt{6}} h = \tfrac{1}{3\sqrt{6}}(150 + 2h),$$

$$\chi = 5 + 2\sqrt{h^2 + (\tfrac{1}{3\sqrt{6}})^2 h^2} = 5 + 2.00111\, h$$
$$= \tfrac{1}{3\sqrt{6}}(150 + 60h),$$

$$\omega = \frac{h}{2}(5 + x) = \frac{h}{30}\left(\frac{150 + 30x}{2}\right)$$
$$= \frac{h}{30}(150 + h),$$

$$v = \frac{Q}{\omega} = \frac{30}{\omega}.$$

The expression for ds may therefore be put under the form

$$ds = \frac{30\,\omega - \frac{v^2}{2g}(300 + 4h)}{(150 + 60h)(av + bv^2)}\,dh.$$

96. In order to integrate it approximatively I construct the subjoined Table, which contains as follows.

1st column. Successive values of h in a series of equidifferences, the ratio being 0.1, and beginning with the given height $h_0 = 2$.

2d column. Values of $30\,\omega = h\,(150 + h)$ in a series the second difference of which is constant, and equal to 0.02.

3d column. Values of $v = \frac{30}{\omega}$ deduced from the preceding column.

4th column. Values of $\frac{v^2}{2g}$, or heights due to the velocities v of the preceding column.

5th column. Values of $300 + 4h$. A series of equidifferences, the constant ratio being 0.4.

6th column. Values of $\frac{v^2}{2g}(300 + 4h)$, product of the numbers in the 4th and 5th columns.

7th column. Values of the numerator of $\frac{ds}{dh}$, or difference of the numbers in the 2d and 6th columns.

8th column. Values of $150 + 60h$ in a series of equidifferences, whose constant ratio is 6.

9th column. Values of $av + bv^2$, corresponding to the values of v contained in the 3d column.

We can deduce them by simple proportion from the numbers already calculated in the Table of Article 29. *

10th column. Product of the numbers of columns 8th and 9th, or values of the denominator of $\frac{ds}{dh}$.

11th column. Quotients of the numbers in the 7th column, divided by those in the 10th, or values of $\frac{ds}{dh}$.

* It might be remarked that the values of $av + bv^2$ are in this Table called values of RI. It is true that in the case under consideration, we cannot have the equation $av + bv^2 = RI$. But a moment's reflection will show that these values of $av + bv^2$ are calculated merely according to the values of a, b, and v, and are the same whether the equation holds good or not. Eytelwein's values are taken rather than Prony's, because the former author had the advantage of a greater number of experiments in determining his coefficients a and b.

TABLE I.

1	2	3	4	5	6	7	8	9	10	11	12
h.	$30\,\omega = h(150+h)$	$v = \frac{30}{\omega} = \frac{900}{30\,\omega}$	$\frac{v^2}{2g}$ = height due to velocity v.	$300+4h$.	Product P of 4th by 5th column.	Numerator of $\frac{ds}{dh}\,30\,\omega - P$. 2d—6th col.	$30\,\chi$ or $150+60h$	$av+bv^2$ from the Tables.	Denominator of $\frac{ds}{dh}$ or prod. of 8th and 9th.	$\frac{ds}{dh}$ or quot. of 7th by 10th.	Distances of consecutive depths.
2.0	304.00	2.960	0.13167	308.0	41.942	262.058	270	0.0010481	0.28299	926.0	
											99.78
2.1	319.41	2.818	0.12335	308.4	38.042	281.368	276	0.0009531	0.26306	1069.6	
											114.63
2.2	334.84	2.688	0.11225	308.8	34.662	300.178	282	0.0008703	0.24542	1223.1	
											130.48
2.3	350.29	2.569	0.10256	309.2	31.712	318.578	288	0.0007978	0.22977	1386.5	
											147.26
2.4	365.76	2.461	0.09407	309.6	29.124	336.636	294	0.0007346	0.21597	1558.7	
											165.02
2.5	381.25	2.361	0.08658	310.0	26.840	354.410	300	0.0006783	0.20349	1741.7	
											183.84
2.6	396.76	2.268	0.07994	310.4	24.815	371.945	306	0.0006281	0.19220	1935.2	
											203.58
2.7	412.29	2.183	0.07404	301.8	23.010	389.280	312	0.0005840	0.18221	2136.5	
											224.23
2.8	427.84	2.104	0.06875	311.2	21.396	406.444	318	0.0005443	0 17309	2348.2	
											245.90
2.9	443.41	2.030	0.06401	311.6	19.945	423.465	324	0.0005086	0.16479	2569.8	
											268.63
3.0	459.00	1.961	0.05973	312.0	18.637	440.363	330	0.0004761	0.15711	2802.8	
											1783.35

TABLE II.

1	2	$v=\frac{660}{30\omega}$	4	5	6	7	8	9	10	11	12
2.9	443.41	1.488	0.03442	311.6	10.726	432.684	324	0.0002827	0.09159	4723.9	452.52
2.8	427.84	1.543	0.03697	311.2	11.506	416.334	318	0.0003026	0.09623	4326.6	
2.7	412.29	1.601	0.03981	310.8	12.374	399.916	312	0.0003244	0.10121	3951·2	418.89

TABLE III.

1	2	$v=\frac{300}{30\omega}$	4	5	6	7	8	9	10	11	12
2.75	420.06	0.713	0.00790	311.0	2.458	417.602	315	0.0000731	0.02303	18135.7	880.32
2.70	412.29	0.728	0.00823	310.8	2.557	409.733	312	0.0000769	0.02399	17077.3	

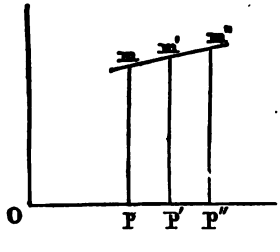
97. We will now show the manner of using the Table, and of finding column 12th. The equation of Article 95 is of the form $ds = F(h)\,dh$. The eleventh column gives the values of $F(h)$ corresponding to the various values of h which are in the first column, and which make a series of equidifferences. Now it may be remarked that these values of $F(h)$ are in a series which does not vary much from a series of equidifferences; whence we may conclude that the distance $S_{m+1} - S_m$, which separates the two consecutive heights h_m, h_{m+1}, is, with considerable approximation, equal to the product of the half sum of the two corresponding values $F(h_m)$, $F(h_{m+1})$, multiplied by the difference $h_{m+1} - h_m$. *

Hence we have a method of determining with a sufficient approximation the length of the canal included between those points where the depth has the various values given in the 1st column; a method which is of very easy application, since the difference between the consecutive values of

* Our notions upon the quadrature of curves render this proposition evident. Imagine a curve whose co-ordinates are h and $F(h)$; the integral $\int F(h)\,dh$ is represented by the area of a segment contained between this curve, the axis of the abscissas h, and the two ordinates which

h is 0.1. By adding together the intermediate results we obtain the total distance between any two depths. Thus h_m being 2 and h_{m+1} being 2.1 we have $h_{m+1} - h_m = 0.1$. We have also $F(h_m) = 926$; $F(h_{m+1}) = 1069.6$; $F(h_{m+1}) + F(h_m) = 1069.6 + 926 = 1995.6$;

correspond to the abscissas which limit the integral.



$Op = h_m$. $Op' = h_{m+1}$.
$pm = F(h_m)$.
$p'm' = F(h_{m+1})$.
$pp' = Op' - Op$
$= h_{m+1} - h_m$.
$\int_{h_m}^{h_{m+1}} F(h)\,dh =$ area $pmm'p'$.

Now if to three equidifferent values of h correspond three values of $F(h)$ sensibly equidifferent, it is clear that the arc which passes through the three points determined by these co-ordinates, differs very little from a straight line. *A fortiori* may we consider the arc which passes through two points as coinciding with its chord; hence the area which is equal to the integral $\int_{h_m}^{h_{m+1}} F(h)dh$ may be taken for that of the trapezoid whose measure is $pp'\left(\frac{pm + p'm'}{2}\right)$. Hence

$$\int_{h_m}^{h_{m+1}} F(h)\,dh = \tfrac{1}{2}[F(h_m) + F(h_{m+1})](h_{m+1} - h_m).$$

and the distance between the sections where the depths are 2 feet and 2.1 is

$$\tfrac{1}{2}\left\{F(h_{m+1})+F(h_m)\right\}(h_{m+1}-h_m)$$
$$=\frac{1995.6}{2}\times 0.1=99.78.$$

In like manner the distance between the sections whose height is 2.1 and 2.2 is,

$$\tfrac{1}{2}(1069.6+1223.1)\ (0.1)=114.63,$$

and so on. The distance therefore between the sections where the depths are 2 and 2.2 is $99.78+114.63=214.41$.

We find also, that between the depths 2 and 2.90 there is a length of 1514.72 feet, and between the depths 2 and 3 feet, a distance of 1783.35 feet.

98. The value of h, therefore, at the end of 1600 feet, which we have supposed to be the length of the canal, will fall between 2.90 which corresponds to a distance of 1514.72 feet, and 3 which corresponds to 1783.35 feet, and must be at a distance of 85.28 from the place where $h=2.90$. We may now remark, that the values of $\frac{ds}{dh}$ corresponding to $h=2.90$ and $h=3$ feet being 2569.8 and 2802.8, the difference of which is 233, it is certain that for values of h increasing by 0.01 from 2.90 upward, the values of $\frac{ds}{dh}$ would increase very nearly by $\frac{2}{1}\frac{3}{0}\frac{3}{0}$ or 23.3.

Thus we should have

$h =$ 2.90 2.91 2.92 2.93 2.94

corresponding to

$\frac{ds}{dh} =$ 2569.8 2593.1 2616.4 2639.7 2663.0,

between which the partial distances are

25.81 26.04 26.28 26.51.*

The sum of the first three partial distances is 78.13. Hence the distance of the point where the depth is 2.93 is 1514.72 + 78.13 = 1592.85 or nearly 1600 feet from the extremity. The answer, therefore, to the question of Article 94 is, that the end where the water is supplied, and which is 1600 feet from the outlet, the depth is 2.93 feet nearly.

99. We will now repeat in a single Article merely what is necessary for a practical application of the rules which we have been explaining in the last five Articles.

We have given

l . . . the length of a canal horizontal at bottom,

w . . . its width at bottom,

Q . . . the discharge per second,

* These partial distances are found in the same manner as those at the end of Article 97;

$$\tfrac{1}{2}\left\{2593.1 + 2569.8\right\}(2.91 - 2.90) = \frac{6162.9}{2} \times 0.01.$$

$$= 25.81.$$

n . . . the ratio of the base of the slope of the sides to their height,

h_0 . . . the value of h, or the depth of the water at the outlet of the canal.

We denote by

x . . . the width of the water at the surface ($x = w + 2nh$),

ω . . . the area of the section ($\omega = \frac{h}{2}(w + x)$),

χ . . . that part of the perimeter which is in contact with the water. ($\chi = w + 2h\sqrt{1+n^2}$).

We wish to know

h_m . . . the depth of water at the entrance of the canal, or at the distance l from the outlet,

v . . . the mean velocity ($v = \frac{Q}{\omega}$).

We construct a table of twelve columns like Table I, containing as follows: *

$$* \frac{\omega - \frac{v^2 x}{g}}{\chi(av + bv^2)} = \frac{\frac{\omega}{n} - \frac{v^2}{2g}\frac{2x}{n}}{\frac{\chi}{n}(av + bv^2)}$$

$$= \frac{h\left(\frac{w}{n} + h\right) - \frac{v^2}{2g}\left(\frac{2w}{n} + 4h\right)}{\frac{1}{n}(w + 2h\sqrt{1+n^2})(av + bv^2)}.$$

Column 1. Successive values of the depth, beginning with h_0 and increasing by small degrees; tenths of a foot, for instance.

Column 2. Values of $\frac{\omega}{n}$, or of its equal $h\left(\frac{w}{n}+h\right)$, found by substituting for h the successive values of column 1.

Column 3. Values of $v = \frac{Q}{\omega}$ or of $\frac{Q}{n h\left(\frac{w}{n}+h\right)}$, found by substituting for h the successive values of column 1.

Column 4. Values of $\frac{v^2}{2g}$ for the successive values of v in the third column. They may be taken from the Table of Article 60.

Column 5. Values of $\frac{2w}{n} + 4h$, found by substituting for h the successive values of column 1.

Column 6. The product of 4th by 5th column.

Column 7. Remainder of the subtraction of the 6th from the 2d column.

Column 8. Values of $\frac{\chi}{n}$ or of $\frac{1}{n}\left(w + 2h\sqrt{1+n^2}\right)$, found by substituting for h the successive values of the 1st column.

Column 9. Values of $a v + b v^2$, corresponding to the successive values of v in the 3d column. They may be found already calculated in the

Table of Article 29, in which they are called values of RI.

Column 10. Product of 8th by 9th column.

Column 11. Quotient of 7th column divided by the 10th.

Column 12. The numbers of this column are thus found. Multiply the half sum of the first two numbers of the 11th column by the constant difference of the successive values of h in the 1st; this gives the first number of column 12.

Multiply the half sum of the second and third number of column 11th by the same constant difference; it gives the second of column 12th and so on. The numbers of column 12 denote successively the distances between the first and second depths, the second and third, and so on.

This will be more clearly understood by referring to Fig. 3. Plate I. and comparing it with Table I. We there see plainly what the numbers of the 1st and 12th columns denote, and that by means of them we are able to draw the longitudinal profile of the canal. Now, in order to find the height h_m at the distance l from the outlet, we go from A towards B, adding up the distances on the line AB until the sum is equal to l, and the height h_m is that corresponding to the point of the line AB where we stop. Thus, in the case of Fig. 3., if $l = 344.89$, the sum

of the first three distances is equal to l, and $h_m = 2.30$. If we had $l = 400$, h_m would fall between 2.30 and 2.40; and, as the surface between these two heights may be regarded as straight, h_m may be found by a simple proportion.

There is no need, however, of constructing the figure; for by merely summing up enough of column 12th to give us a sum equal to l, the corresponding height will be found in the 1st column.

100. M. Bélanger remarks that we should not use these formulæ when the values of h are such as to give to $\frac{ds}{dh}$ values less than 30; because in that case the approximation to a parallel motion, which we have supposed in the different linear elements of the stream, may no longer take place in a sufficient degree for us to reason upon this supposition.

101. Thus far we have supposed, that all the water which flows in the canal is discharged by an outlet at the extremity. But, if the object of the canal be to furnish a supply to pipes, which are connected with it in various parts of its length, this will give rise to another problem which we will now proceed to solve.

102. Suppose then that the same canal of which we have been speaking, discharges 8 cubic feet per second by a pipe which opens into it at a distance of 300 feet from the extremity

where the water is supplied; 12 cubic feet by a pipe 600 feet from the first; and the remainder, 10 cubic feet, by the outlet at its extremity. So that it conveys

for a length of 300 feet 30 cubic feet per second
" " 600 " 22 " "
" " 700 " 10 " " .

We suppose, moreover, that the water at the beginning of the canal is 3 feet deep. The question is, what will be the inclination of the surface in the three different portions of the canal.

103. The first division, with a length of 300 feet, is to discharge, as in the former question, 30 cubic feet; we can use then Table I. of Article 96, relating to that question. We see that the distance between the depths 3 feet and 2.80 is 514.53 feet; there being 245.90 between 2.80 and 2.90, and 268.63 between 2.90 and 3. The value of h corresponding to 300 feet will therefore be contained somewhere between 2.80 and 2.90; and may be found by a process similar to that of Article 98.

The values of $\frac{ds}{dh}$ being 2348.2 and 2569.8, the difference of which is 221.6, the difference corresponding to a variation of 0.01 of a foot in the value of h, will be, with a sufficient approximation, 22.16. Thus we shall have

h =	2.80	2.81	2.82	2.83	2.84	2.85
$\frac{ds}{dh}$ =	2348.20	2370.36	2392.52	2414.68	2436.84	2459.00
distances =	23.59	23.81	24.03	24.25	24.48	

h =	2.85	2.86	2.87	2.88	2.89.
$\frac{ds}{dh}$ =	2459.00	2481.16	2503.32	2525.48	2547.64.
distances =	24.70	24.92	25.14	25.36.	

The sum of these nine partial distances is 220.28. The point at which we want the depth is 214.53 from the place where $h = 2.80$. Hence this depth will be very nearly 2.89. Hence we conclude that the fall of the surface from the entrance of the canal to the origin of the first pipe is 3 — 2.89 or 0.11 in a length of 300 feet.

104. The second division is to discharge the quantity 22 cubic feet; we therefore have $v = \frac{22}{\omega} = \frac{660}{30\,\omega}$. We must now form a Table similar to Table I, the titles of the several columns being the same as those of the preceding Table, with the exception of the 3d, in which v has the above value. For greater convenience we will begin with $h = 2.90$, and will afterwards make the proper correction.

With only three lines of this Table we see that the distance between the depths 2.90 and 2.70 is 452.52 + 413.89 or 866.41. In order to know the distance between the ordinates 2.90

and 2.89, we may remark that between the depths 2.90 and 2.80 the quantity $\frac{ds}{dh}$ varies by $\frac{1}{10}$ (4723.9 — 4326.6) = 39.73 for a variation of 0.01 in the value of h.

Thus we have

h = 2.90 2.89,

$\frac{ds}{dh}$ = 4723.9 4684.17,

distance = 47.04.

Hence we conclude that the distance between $h = 2.90$ and $h = 2.89$ is 47.04. Thus between the depths 2.89 and 2.70 there is a distance of 866.41 — 47.04 or 819.37 feet. It remains now to see what will be the depth at a point 819.37 — 600 or 219.37 from the depth h = 2.70, going towards the point where the depth is 2.80.

By the same process as that used before we have

h =	2.70	2.71	2.72	2.73	2.74	2.75
$\frac{ds}{dh}$ =	3951.2	3988 74	4026.28	4063.82	4101.36	4138.90
distances =	39.70	40.07	40.45	40.83	41.20.	

The sum of the first five distances is 202.25. Hence we may say with considerable approximation, that at a point 219.37 from the place where h = 2.70, we shall have h = 2.75.

Hence we conclude for the second division, that the depth at the origin of the second dis-

charging pipe is 2.75, which makes a fall of 2.89 — 2.75 = 0.14 feet in the length of 600.

105. Remains now to consider the third division. In this we have $v = \frac{10}{\omega} = \frac{300}{30\,\omega}$, and with this value of v we calculate Table III, in the same manner as we did Table II, beginning with $h = 2.75$ and increasing the values of h by 0.05 at a time, on account of the small velocity, which renders $\frac{ds}{dh}$ very great. By this Table we see that the depth 2.70 will be at a distance 880.32 from the depth 2.75. The point in question, at which we wish to know the depth, is therefore at 180.32 from where the depth is 2.70, in going towards the depth 2.75. Using the same process of calculation as before, we shall have

$$h = 2.70 \qquad 2.71,$$

$$\frac{ds}{dh} = 17077.3 \qquad 17289.0,$$

$$\text{distance} = \qquad 171.83.$$

Hence we may conclude with a sufficient approximation, that at a point 700 feet from where $h = 2.75$, we shall have $h = 2.71$; and therefore that the fall in this third division is 2.75 — 2.71 = 0.04 in 700 feet.

106. Recapitulating the results which we have obtained, we have

	Depth of water.	Fall in each division.	Length of each division.	Quantity which flows through each division.
At the entrance of the Canal, where the quantity discharged is, per second, 30 cubic feet	3 feet	ft.	ft.	
Entrance of first pipe where quantity discharged is 8 cubic feet	2.89	0.11	300	30 cubic feet
Entrance of second pipe where the discharge is 12 cubic feet	2.75	0.14	600	22 "
At outlet where the discharge is 10 cubic feet	2.71	0.04	700	10 "

By means of these Tables we can now construct Figure 4. Plate I., showing the longitudinal profile of the canal, in the same manner as we did that relating to the first question.

107. M. Bélanger here remarks, that, in solving this last question, he has supposed without any proof, in passing from one division to another: 1st, that the first section of each division has the same height as the last of that which preceded it, although the velocities are not the same; 2dly, that the drawing off a portion of the water by branch pipes or aqueducts would not interfere with the motion, which was supposed to be in sensibly parallel lines, in all of which the water passes any section with the same velocity. In the case of great velocities this would undoubtedly be inadmissible; for in that case there would certainly be on the surface, immediately after each branch pipe, a sensible rise in the stream,

or an inclination in the direction opposite to the general one, which would be caused by the great and sudden diminution of velocity. But he thinks his two hypotheses may be admitted in the case of small velocities, in which the error cannot certainly but be very slight.

108. The method, which we have just been explaining, for finding the conditions of the motion of a stream of water in a channel whose bottom is horizontal, has not been tried by experiment. As a theoretical method, it is of easy application, if we are willing to be satisfied with an approximation fully sufficient for practical purposes. The construction of the Tables is much simpler than might at first be imagined. It is much to be desired, however, that accurate and varied experiments should be made, in order to confirm these results of analysis, or to show wherein they are deficient. But until then, although a practical demonstration is wanting, we must consider them, in the absence of experiment, as our safest guide.

CHAPTER VI.

CONVEYANCE OF WATER BY CONDUIT-PIPES.

109. By means of the formulæ and Table of Chapter II. we shall be able to solve with great facility the questions relating to the conveyance of water by Conduit-Pipes. Some examples will best show how to apply them.

We wish to know what will be the velocity and the quantity of water per second conveyed 8 miles, by a pipe of 1 foot interior diameter; the fall from the fountain-head to the surface of the water in the reservoir being 50 feet.

We have

$$D = 1,$$
$$\zeta = 50,$$
$$\lambda = 42240,$$
$$j = \frac{50}{42240} = 0.0011837 = Dj,$$
$$\tfrac{1}{4} Dj = 0.0002959.$$

By Table I. of Article 44, we see that the velocity will be a little less than 1.60, which corresponds to 0.0002995. We can find it with sufficient accuracy by a simple proportion. Taking the difference between the values corresponding to 1.55 and 1.60, we have

$$176 : 0.05 : : 140 : 0.04.$$

Hence $v = 1.55 + 0.04 = 1.59$.

To find the quantity per second we have $Q = \frac{v \pi D^2}{4} = 0.7854 \times 1.59 = 1.25$ cubic feet.

Instead of using the Table we might have used the equations of Article 43 for the velocity or the quantity. In the equation

$$0.000088268\, D^2 Q + 0.0006881319\, Q^2 = D^5 j$$

we can substitute for D and j their values and we shall have

$$0.000088268\, Q + 0.0006881319\, Q^2 = 0.0011837,$$

whence we deduce $Q = 1.25$.

110. Generally, however, the known quantity is the supply; and the question is, what are the dimensions requisite to convey a certain number of cubic feet per day. Suppose, for instance, we need 500,000 cubic feet per day, or 5.787 cubic feet per second, the distance and the fall being the same as in the preceding case; what will be the requisite diameter of the pipe?

We might substitute for D and j their values in the third equation of Article 43; but we shall use in this case the second Table of Article 44, which will give the same result as the fifth equation.

We find by a proportion the value of $D^5 j$ corresponding to $Q = 5.787$, which will fall be-

tween 0.0227575 and 0.0231550. We have

$$0.05 : 3975 : : 0.037 : 2941;$$

$$D^5 j = 0.0227575 + 0.0002941 = 0.0230516;$$

$$D = \sqrt[5]{\frac{0.0230516}{0.0011837}} = 1.81 \text{ feet.}$$

111. It may happen that the ground on which the city stands is uneven, and that the height of the source, whence the water is drawn, may not be sufficient to raise it to the more elevated spots. If we have only a fixed sum to lay out, which sum may be sufficient to purchase and lay down a pipe of a certain diameter, we may make the pipe discharge into a reservoir placed at some moderate height, in which case it will furnish a certain determined quantity of water, or place the reservoir at a height considerably greater, in which case the quantity of water will be much less, but the number of houses within reach of the supply will be greater. The most proper course to adopt in such a case would be, to discharge the water in the city at a height such that the quantity will be just sufficient to supply all the houses below this height. If we take a higher point, our quantity will fall short; and if we take a lower point, we shall be unable to reach a sufficient number of houses to make use of all that we are able to furnish. Of course, if it should happen that the lower parts of the city

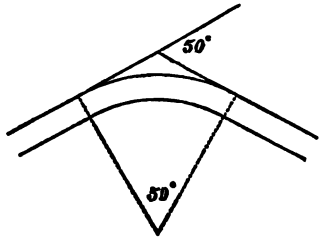
comprise principally the dwellings of the poorer classes, while those of the richer inhabitants, who would be willing to pay a greater sum for the luxury of good water, are generally situated on the higher ground, this circumstance would have an influence on our calculations, and lead us to place the point of discharge at a higher level than we otherwise should.

112. Suppose, for instance, that we are to use a pipe of 1 foot diameter and 20,000 feet long. In this case, in Table II. of Article 44 we have $D^5 = 1$, and the column $D^5 j$ contains simply the values of j or of $\frac{\zeta}{\lambda}$, or of $\frac{\zeta}{20{,}000}$. Thus a discharge of 1 cubic foot per second corresponds to a head of

$$0.0006883 \times 20000 = 13.766;$$

1.25	cubic feet	to a	head of	21.510,
1.50	"	"	"	30.974,
1.75	"	"	"	42.160,
2	"	"	"	55.066.

113. It may happen that, in laying down the pipe mentioned in Article 109, we shall find it necessary to make a bend such that one branch may deflect 50 degrees from the other. We desire to know the proper radius for the curve which joins the two branches, and the height of water necessary to overcome the resistance at the bend.



In that case we have (see Art. 47.)

a . . . angle of reflection;

α . . . angle of deflection or at centre $= 50°$;

D . . . $= 1$;

$$\cos a = 1 - \frac{1}{2r};$$

$$n = \frac{50°}{2a} = \frac{25}{a}.$$

In order that the water may not be irregularly reflected against the sides of the pipe, we must give to a such a value as will render n a whole number. If we suppose $a = 5°$ we shall have

$$n = 5;$$

$$\cos 5° = 1 - \frac{1}{2r};$$

$$r = \frac{1}{2\,(1 - \cos 5°)} = \frac{1}{0.00762} = 131.23 \text{ feet.}$$

We might have made $a = 25°$, which would give $n = 1$, and this would also cause a regular reflection into the axis of the second branch. We should then have

$$r = \frac{1}{2\,(1 - \cos 25°)} = \frac{1}{0.18738} = 53.37.$$

Thus we see that we may make the radius of curvature equal to 131.23 feet or to 53.37, and

in either case the water would be reflected in a regular manner from the axis of one branch into that of the other. We might have also made $a = 2^\circ\ 30'$ which would have given $n = 10$.

The height requisite to overcome the resistance arising from the bend is $0.003755\ v^2 s^2 n$. (Art. 47.)

If we make $a = 5^\circ$ we have $n = 5$,

$$s = \sin 5^\circ,$$
$$v = 1.59,$$

and the height given by the above formula is 0.00036 foot.

If we make $a = 25^\circ$, we have $n = 1$,

$$s = \sin 25^\circ,$$
$$v = 1.59,$$

and the height given by the above formula is 0.00170.

Thus we see that there is considerable advantage in using the larger radius 131.23.

According as we adopt one or the other radius, we should deduct the corresponding height from the head ζ, and use the remainder for the value of ζ in the general formula for finding the discharge.

114. A question may arise as to the manner in which water may be drawn from two reservoirs at a time; and the preceding rules will enable us to solve it without difficulty.

We have two ponds A and B, situated at unequal distances from the city which we wish to supply with water, and at unequal heights. The question is, how can we take advantage of both of them; supposing them nearly in a straight line from the point at which we are to deliver the water.

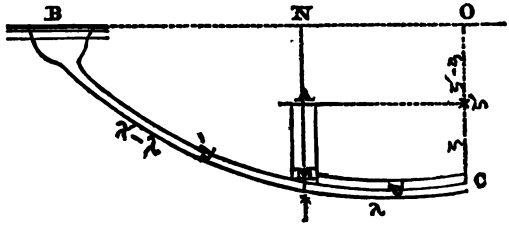
Let q = quantity per second which A can furnish;

ζ, λ, the fall and the length from A to the discharging orifice.

Let q' = quantity per second which B can furnish;

ζ', λ', the fall and the length from B to the discharging orifice.

The question may be solved in the following manner.



Suppose a reservoir in B, whence water flows by a pipe to the discharging orifice C. The pressure at M is equal to the weight of a column of water of a certain unknown height

AM.* This pressure is supported by the side of the pipe. Evidently we do not change any thing in the conditions of the motion, if we take off a portion of the upper side of the pipe at M, and replace it by a reservoir of water AM. It is evident also that AN, or that part of the head lost between B and M, is the portion employed to overcome the friction between B and M, just as OC is the head employed to overcome the friction between B and C. Things being in this state, if we imagine a division on the line AM, it is evident that on the left of the line a certain quantity of water is arriving from B into the reservoir A, and that on the right of the line an exactly equal quantity is flowing out from the reservoir to C.† The whole of the water is now furnished by the reservoir at B, it is true; but the

* What the pressure is, and consequently what this height AM will be, is shown in Articles 162 and following.

† The height necessary to overcome the friction in the same pipe is proportional to the length. Hence AN or $\zeta' - \zeta$ being the head between B and A, and ζ or $OC - AN$ the head between A and C, the pipe having throughout the same diameter d', we have

$$\zeta' - \zeta : \lambda' - \lambda :: \zeta : \lambda,$$

or $\frac{\zeta' - \zeta}{\lambda' - \lambda}$, which is the value of j between the res-

case is exactly the same as if AM were a solid partition, and the quantity q' were supplied by the reservoir at A, the latter maintaining a constant level.

We will now suppose that by means of certain supplies of its own, the reservoir A is capable of furnishing of itself a quantity q without its level being affected. It is capable also of furnishing the additional quantity q', which it receives from B. Hence if we give to the pipe MC a diameter d, capable of discharging $q + q'$, determined by the equation

$$q + q' = c\sqrt{d^5 \frac{\zeta}{\lambda}},$$

we evidently make no change in the conditions of the motion between B and M. At the same time, since the quantity q' is to be conveyed from B to A, the diameter d' of the pipe between the two reservoirs must be determined by the equation

$$q' = c\sqrt{d'^5 \frac{\zeta' - \zeta}{\lambda' - \lambda}}.$$

ervoirs, is equal to $\frac{\zeta}{\lambda}$, which is the value of j between A and C. Hence the diameter and the values of j in the two parts being the same, the quantities determined by $q' = c\sqrt{d'^5 j}$ must be the same.

The diameters d and d' determined in the preceding manner are evidently the proper ones in the case under consideration, and any other diameters would be attended with disadvantage. For if d' were made greater, it would convey $Q > q'$; and the reservoir A having means only to discharge q' besides its own proper supply, its level would be raised until it had attained a sufficient head to discharge $Q + q$, in which case B would lose more water than it can spare according to the conditions of the question. Or if, instead of allowing the level to be raised we increased the diameter d, B would also lose more than it can spare. If, again, peculiar circumstances, as a dam or overfall for instance, prevented the level A from being raised, the excess of Q over q' would flow off at A as from a leak, and be wasted. Nor can we make d' smaller than we have determined it to be; for in that case it would not convey as much water as we are able to furnish.

Also, if d were greater or less than we have determined it to be, it would discharge more than, or not so much as, A and B together can furnish.

115. It may be asked what would be the event if the sides of the reservoir A were raised, and the supplies by means of which it is enabled to furnish its own quantity q were increased, so that it could furnish $Q > q$.

(1.) If we increased the diameter of the pipe MC in the proper ratio, the discharge at C would be increased by a quantity equal to the additional supply. No other effect could be produced.

(2.) If nothing was changed in the pipes, the reservoir A receiving more than it could discharge, its level would be raised. This evidently alters the head between B and A, and would therefore affect the motion between the reservoirs; and also the head between A and C, and would therefore affect the motion between A and the point of discharge. The level of A being raised, and that of B remaining the same, the quantity flowing from B to A must be $q'' < q'$ on account of the diminished head. The supplies received by A are therefore $Q + q''$, and its level will become constant when d is capable of discharging this quantity. Let ζ'' represent the head between the new surface in A and the point of discharge C. Evidently we shall have

$$Q + q'' = c\sqrt{d^5\frac{\zeta''}{\lambda}}.$$

But the difference of level between B and A is $\zeta' - \zeta''$. Hence

$$q'' = c\sqrt{d^5\frac{\zeta' - \zeta''}{\lambda' - \lambda}}.$$

From these two equations we deduce *

* $Q + q'' = c\sqrt{d^5\frac{\zeta''}{\lambda}}$.

$$q''^2\left(1+\frac{d^5}{d'^5}\frac{\lambda'-\lambda}{\lambda}\right)+2Qq''=c^2\frac{d^5}{\lambda}\zeta'-Q^2.$$

Making

$$1+\frac{d^5}{d'^5}\frac{\lambda'-\lambda}{\lambda}=A. \qquad \frac{c^2d^5}{\lambda}\zeta'=B.$$

$$q''=-\frac{Q}{A}+\sqrt{\frac{Q^2}{A^2}+\frac{B}{A}-\frac{Q^2}{A}}.$$

q'' being thus found, we have from the 1st equation

$$\zeta''=\frac{(Q+q'')^2\lambda}{c^2d^5}.$$

This shows what will be the height of the reservoir at A, and what will be the quantity conveyed from B to A.

$$q''=c\sqrt{d'^5\frac{\zeta'-\zeta''}{\lambda'-\lambda}}; \qquad \frac{q''^2(\lambda'-\lambda)}{c^2d'^5}=\zeta'-\zeta''.$$

$$\zeta''=\zeta'-\frac{q''^2(\lambda'-\lambda)}{c^2d'^5}.$$

$$Q^2+2q''Q+q''^2=\frac{c^2d^5}{\lambda}\zeta'-\frac{c^2d^5}{\lambda}\frac{(\lambda'-\lambda)}{c^2d'^5}q''^2.$$

$$q''^2+\frac{d^5}{d'^5}\frac{(\lambda'-\lambda)}{\lambda}q''^2+2Qq''=\frac{c^2d^5}{\lambda}\zeta'-Q^2.$$

$$q''^2\left(1+\frac{d^5}{d'^5}\frac{(\lambda'-\lambda)}{\lambda}\right)+2Qq''=\frac{c^2d^5}{\lambda}\zeta'-Q^2.$$

$$q''^2A+2Qq''=B-Q^2.$$

$$q''^2+\frac{2Q}{A}q''=\frac{B-Q^2}{A}.$$

$$q''=-\frac{Q}{A}+\sqrt{\frac{Q^2}{A^2}+\frac{B}{A}-\frac{Q^2}{A}}.$$

116. It may be asked, what is the greatest height at which water from the reservoir A can be delivered; or in other words, what is the least possible fall or value of ζ between A and C. This will be given by the formula

$$q = c\sqrt{d^5 \frac{\zeta}{\lambda}},$$

whence

$$\zeta = \frac{q^2 \lambda}{c^2 d^5}.$$

Giving determinate values to q and d, we shall have ζ.

Let q_1 be the least value which we can give to the quantity, and D the greatest admissible value of the diameter. We shall have for the least value of ζ, which corresponds to the greatest height,

$$\frac{q_1^2 \lambda}{c^2 D^5} = \zeta.$$

It may be asked, what influence will a reservoir in B have. Will it enable us to raise the same quantity of water higher, or to raise a larger quantity as high?

If the water from B can raise the level of A any quantity h, it will evidently raise the point of discharge just as much; for ζ will in that case be measured down from the new surface. If the level A cannot be raised, and B only furnishes a supply, the water brought from B must run to

waste, and can have no effect upon the height or quantity. For we can only increase the quantity or diminish the fall, 1st, by increasing D; but we have already supposed D as large as possible; or, 2dly, by exerting an additional pressure in M against the water flowing to C. But this we cannot do, in the present instance; for the column AM, whatever may be the motion among the particles of water of which it is composed, must always exert exactly the same pressure, as long as it retains the same height; and we have supposed the height invariable.

117. Let us suppose, in the case of Article 114,

$q = 3$ cubic feet, $\quad q' = 5$ feet,
$\zeta = 20$ feet, $\quad \zeta' = 50$ feet,
$\lambda = 18{,}000$ feet, $\quad \lambda' = 40{,}000$ feet.

What are the diameters d' and d?

$$q + q' = c\sqrt{d^5 \frac{\zeta}{\lambda}}, \quad q + q' = 8, \quad \frac{\zeta}{\lambda} = \frac{1}{900}.$$

The second Table of Article 44 gives for a discharge of 8 feet,

$$d^5 \frac{\zeta}{\lambda} = 0.0440523.$$

Hence

$$d^5 = 0.0440523 \times 900, \quad \text{and} \quad d = 2.0876;$$

also

$$q' = c\sqrt{d'^5 \frac{\zeta' - \zeta}{\lambda' - \lambda}}; \quad q' = 5; \quad \frac{\zeta' - \zeta}{\lambda' - \lambda} = \frac{3}{2200}.$$

From the Table we have, for a discharge of 5 feet,

$$d'^5 \frac{\zeta}{k} = 0.0172080.$$

Hence

$$d'^5 = \frac{0.0172080 \times 2200}{3}, \quad \text{and} \quad d' = 1.6604.$$

118. Let us suppose in the case of Article 115, that $q = 3$ becomes $Q = 4$, all other things remaining the same. What will be the quantity q'' conveyed from B to A? What will be the new level of the surface at A?

$$A = 1 + \left(\frac{2.0876}{1.6604}\right)^5 \frac{22000}{18000} = 4.84,$$

$$B = \frac{(38.116)^2\,(2.0876)^5 \times 50}{18000} = 160,$$

$$q'' = -\frac{4}{4.84} + \sqrt{\left(\frac{4}{4.84}\right)^2 + \frac{160}{4.84} - \frac{4^2}{4.84}} = 4.6896,$$

$$\zeta'' = \frac{(8.6896)^2\,18000}{(38.116)^2\,(2.0876)^5} = 23.60.$$

The quantity conveyed from B to A is 4.6896 cubic feet. The whole quantity discharged is 8.6896. The reservoir A is raised 3.60 feet.

119. As the great resistance to the motion of water in pipes, arising from the friction against the sides, cannot in any way be avoided, it is the more necessary, when water is to be conveyed by this method from a considerable distance, to remove every other obstacle to its pas-

sage. A perfectly straight line is the best direction which can be given to the pipe; but, as it is generally impracticable to do so, the nearer approach we make to it the better. All sudden turns should be carefully avoided. The curves, either in a horizontal or vertical plane, should be made of as large a radius as possible. Embankments and excavations to a considerable extent should be encountered, in order to preserve the line. For it should be recollected that every obstruction to the discharge, will demand a corresponding increase in diameter; and this increase, though slight, yet, taking effect on the whole length, may occasion a considerable expense.

120. When a pipe has vertical flexures, the air which finds its way into it, will rise and occupy the summit of these flexures, and, unless means are taken to get rid of it, will diminish, or even wholly obstruct the flow. Couplet found, that when water was admitted into a pipe which leads from Roquencour to Versailles, about 12,000 feet long and eight inches in diameter, with a head of a little more than two feet and a half, about ten days elapsed before it made its appearance at the mouth. In consequence of this, some of the curves were made less abrupt, and the summits of the vertical flexures were made to communicate with the atmosphere by means of

small, upright pipes, so that the confined air could escape. After this, the water generally began to flow in about twelve hours, being preceded all this time by gusts of wind, and small irregular discharges of mingled air and water.

121. The method here spoken of for getting rid of the air, seems to be the most simple which can be employed. If a small vertical pipe be inserted upon the summit of one of the curves, the water will rise in it till it has attained the height corresponding to the pressure at that place; the air will pass through it and escape. But when the head of water is considerable, the great height, which it would be necessary to give these pipes, renders them inconvenient.

122. In the pipes which convey water through the streets of Paris, one of the two following methods is used. The first is, to connect with the main a short pipe with a stop-cock. It is opened when the water is first admitted, and shut as soon as the air has all passed out and the water begins to follow. The second is by means of an air valve represented in section in Figure 5. Plate II. A copper cylinder *a*, about 8 inches in diameter on the outside and 14 inches high, is put in communication with the main *c* by a short four-inch pipe *b*. Inside of the cylinder are two cross-pieces with each a hole in the centre, through which passes a stem *f*, which

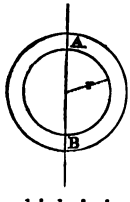
forms the axis of a hollow ball *g*, used as a float. On the upper end of the stem is a portion of a cone *h*, which exactly fits an opening *i* in the cover of the cylinder, when the floating ball is raised by the water. As soon as the air from the pipe enters the cylinder and has acquired a sufficient density to force the water downwards, the float descends with it, and thus leaves an opening in the cover through which the air escapes.

123. Pipes are made of various materials. Wood is often used as being the cheapest; but under a great pressure it will be very liable to burst; and if water is to be supplied to a large city, it is altogether too perishable a material, in a situation where any interruption to the regular supply would be an immense evil. Lead is generally used for small branch-pipes, and is very convenient on account of its flexibility. But the pipes which experience seems to point out as the best are those of cast iron, and these are now almost universally used on lines of importance. They may be made of any strength, will last a very great length of time, and are cheaper than any other material possessing the same advantages.

124. The pressure, which the water exerts against the sides of the pipes through which it flows, is often very great. It is equal in any

point to the weight of the column which would rise in a vertical pipe at the place in question; that is, of a column whose height would be equal to that of the reservoir above this point, diminished by the height required to overcome the friction between this point and the reservoir. (See Articles 163, 164, 165.)

125. In order to estimate the proper diameter to resist a given pressure, we may reason as follows.



Let r be the inner radius of the pipe, and t its thickness, in inches, P the pressure upon a square inch inside of the pipe, R' the tension per square inch to which we may venture to expose the material of which it is made. Suppose the pipe divided by a solid partition AB in the direction of its diameter. This evidently will not change the direction or the intensity of the various forces. Now it is evident, that in the interior of the pipe the whole pressure of the water exerted against this partition will be $2\,r\,P$. On the other hand, the resistance at A and B evidently is $2\,t\,R'$.

Hence, if we make

$$t\,R' = r\,P \qquad t = \frac{r\,P}{R'},$$

the thickness t determined by this equation will

be such that the parts of the pipe will support a tension equal to R' on a square inch.

126. The value of R' must be determined by experiment. According to Mr. G. Rennie, the cohesive force of iron cast horizontally is 18,656 lbs. per square inch, and 19,488 lbs. when cast vertically. From some experiments made by Captain S. Brown, the cohesive force is 18,637 lbs. Mr. Tredgold says that cast iron will not bear without alteration more than 15,300 lbs. per square inch. In general we may observe that this material ought not in practice to be subjected to a strain greater than one fourth of that which would produce a rupture.*

127. In fixing upon the proper thickness for pipes, we should bear in mind that, besides the pressure to which they are subjected when the water is in motion, they are also exposed to violent efforts, when the velocity the water has acquired meets with a check by a sudden closing of the discharging cocks. If made of wood, they are exposed to rot. If of lead, they are liable to be acted upon chemically by matters contained in the earth. If made of iron, they are subject more or less to rust, which impairs their strength.

* Navier, *Résistance des Matériaux.*

Tredgold, *A practical Essay on the Strength of Cast Iron.*

Before being used, the pipes should be proved by means of a forcing-pump, and subjected to a pressure at least three times as great as that produced by the water which they are to convey. The greatest head upon the pipes used in Paris is about 65 feet, and the pressure by which they are proved is 10 atmospheres.

128. Iron pipes cast horizontally are often weaker on the upper side than on the lower. When cast vertically, they are more likely to be of equal strength throughout. It is well to examine them on the very spot where they are to be used, as the jolting, to which they are exposed during their transportation from the foundery, has been sometimes shown by experience to have a prejudicial effect upon the most defective parts, without however producing any apparent crack.

129. A question may arise as to the comparative advantages of employing a single pipe of large diameter, or several of a smaller diameter, to convey the same quantity of water. In the latter case, an accident does not interrupt the supply as in the former.

The quantity of water being determined (Art. 43) by the formula $Q = c\sqrt{j}\sqrt{D^5}$, if, instead of a single pipe of the diameter D, we wish to use a number m of pipes of a smaller diameter d, we can determine the value of d as follows.

$$m\,q = Q, \qquad q = c\sqrt{j}\sqrt{d^5},$$

$$m\,q = m\,c\sqrt{j}\sqrt{d^5},$$

$$c\sqrt{j}\sqrt{D^5} = m\,c\sqrt{j}\sqrt{d^5},$$

$$D^5 = m^2\,d^5,$$

$$d = \frac{1}{\sqrt[5]{m^2}}\,D.$$

Suppose $m = 2$,

$$d = \frac{1}{\sqrt[5]{4}}\,D = 0.758\,D.$$

130. If we wish to know the comparative expense, we may remark that, according to Article 125, the thickness increases as the radius or the diameter. The circumference increases also as the diameter. Hence the section, or the weight, which increases as the circumference multiplied by the thickness, increases as the square of the diameter. Thus, with one pipe, the weight would be proportional to D^2. With m pipes of the diameter d, it would be proportional to $m\,d^2$. The expense, being as the weight, would therefore be in the two cases as $D^2 : m\,d^2$.

But

$$d = \frac{1}{\sqrt[5]{m^2}}\,D.$$

Thus the ratio will be

$$D^5 : \frac{m}{\sqrt[5]{m^4}} D^5,$$

$$\text{or } 1 : \sqrt[5]{m}.$$

Suppose $m = 2$. The expense of a single pipe would be to that of two pipes, as

$$1 : \sqrt[5]{2}, \quad \text{or } 1 : 1.149.$$

131. It is highly important that the short pipes, of which the conduit is composed, should be joined together in the most perfect manner. When they are of wood, it may be done in the manner shown in Figures 5. 6. 7. of Plate I. Figure 5. is the most common. The end of one pipe is enlarged, and the end of another sharpened so as to enter into it. The joint is strengthened by an iron strap *C*. In Figure 6. a ring is made sharp at the edges, and enters the butt of each pipe. Figure 7. shows another kind of joint. With all of them, tow should be carefully stuffed in between the pieces to make them tight.

132. When wooden pipes of considerable diameter are required, they may be made of staves bound with iron hoops. In some instances, planks have been bent by steam into the shape of rings, and joined together at the edges by rings of iron, as shown in Figure 6. This has been found to answer very well, and will be very

strong if the wood is carefully selected. Leaden pipes are generally connected by soldering. Sometimes a flanch or rim is made around the ends, and they are drawn together by means of screws, which pass through holes in the rims, a piece of soft leather being inserted between them. Cast iron pipes may be united in various ways. Figure 8. represents the manner last mentioned for lead, and it has often been used for iron. In order to avoid the necessity of making holes in the rims, the mode shown in Figure 10. has been adopted. When the two parts of the collar *k l*, which embraces them, are drawn together by the screws *n*, the rims, which are wedge-shaped, are forced together. These joints are very strong, but they do not allow the least play, which is a disadvantage. Figure 9. shows a more approved mode of connexion. The end of one pipe being introduced into an enlarged part at the end of the other, tow is first pressed into the opening, and the remainder of the space is then filled with melted lead, which is compressed by hammering. The best mode of stopping up the joint, when lead is to be poured in, is by a ring composed of two semicircular pieces, which are connected by a hinge at one end, and can be screwed together at the other. This ring is made to clasp the inserted pipe close to the joint. The lead is then pour-

ed in, and the ring not only prevents it from running out, but leaves it with a smooth and even surface, so as to render no dressing or chipping necessary around the joint. This mode was used at Philadelphia.

133. The contraction and expansion of iron pipes, occasioned by variations in temperature, would infallibly cause them to break, if means were not taken to allow these variations in length to take place. When pipes are connected by screws in the manner first mentioned, it is necessary to have at certain distances *compensating joints,* which permit the whole variation of length for a considerable distance to produce its effect at one place. Figure 1. Plate II. shows sufficiently the manner in which this may be done. The linear dilatation of cast iron for 180° Fahrenheit is 0.00111 of its length. As within these limits it may be considered as taking place uniformly, we may easily calculate, according to the probable variations of temperature, and the distance apart at which we mean to place these compensating joints, what their play must be. This mode evidently cannot be adopted when the pipes are laid directly in the ground. But if they are connected by such a joint as that of Figure 9. Plate I., no such compensation is necessary. The very little play which there may be at each joint answers the purpose perfectly well;

and accordingly this has been most commonly adopted. The linear dilatation of lead is much greater than that of cast iron, being in the proportion of 28 to 11. On this account there is much greater reason to fear inconvenience from variations in temperature with this metal, than with the other; and in Paris large leaden pipes have of late years been replaced by iron.

134. In order to be able to take out a defective pipe, and replace it, which the kind of joint last mentioned does not permit, there should be at certain distances a butt-joint, covered with a ring of considerable width. If two of these rings are driven aside, the space between them may be taken up and replaced. A broken pipe may sometimes be repaired without any interruption by means of a covering of sheet-lead, the ends of which are screwed tight to the pipe; or by means of two half-cylinders of cast iron, screwed together to enclose it.

135. The most common materials used for caulking the joints, are bits of tarred rope, and lead. The caulking matter ought to have some elasticity, especially where pipes pass under streets, otherwise the jarring occasioned by heavy carriages produces a bad effect.

136. M. Gueymard used, for pipes laid at Grenoble, a cement which becomes at least as hard and compact as good cast iron. Ninety-

eight parts of cast iron filings, free from oxidation, after being sifted through a coarse sieve, are thoroughly mixed with one part of flour of sulphur. One part of common grey sal ammoniæ is then dissolved in boiling water and poured over the other two ingredients, and the whole is well kneaded together.

The quantity of water should be such as to produce a mixture of about the consistence of common mortar. This cement disengages a great quantity of caloric and of ammonia, and should be used immediately. It should only be prepared just as it is wanted. When first put into the joints, it should be driven in tight with a chisel and mallet. It should be left to dry two or three days in the open air in summer, and seven or eight in winter. As it has no elasticity and allows no play at the joints, M. Gueymard found it necessary to use tow instead of it for a good many of them.

A mixture of linseed oil, tow finely chopped, and air-slacked lime, of about the consistence of common putty has sometimes been used.

137. In large cities, where it might often be a very serious evil to interrupt the passage of a street in order to repair a leak, the pipes have sometimes been carried through small galleries, by which means they may at any time be inspected, the defective parts seen immediately,

and replaced or repaired. This mode, however, is so expensive, that it will very rarely be used. They are sometimes placed in a little channel of brick or stone, covered with plank. The disadvantage of this is, that, if there is a leak, the water runs off in this channel, so that it is very difficult to find the place.

138. The most common way is, to lay the pipes directly in the natural ground. They should be placed at least four feet below the surface, in order to avoid the effect of frost, and the jarring to which they are exposed when passing under frequented roads or streets. It has been remarked in Paris, that the pipes in the neighbourhood of the stone-yards, where very heavy loads pass frequently, require repairs much oftener than anywhere else.

The pipes being in place, the water should be let in upon them, and all the joints examined before they are covered.

139. At various parts of the length of a line of pipes, there should be stop-cocks and discharging cocks. By means of the first, the passage of the water may be stopped; while, by the second, the water contained in any part of the pipes may be let off in order to cleanse or repair them. These discharging cocks should be placed at the lowest parts of the vertical flexures, and there should be a drain or channel by which the water, when let out, may run off.

The form of these cocks should be such that, when open, they may not offer any obstruction to the passage of the water; and they should shut slowly, in order to avoid the effect of a sudden check upon the velocity of the water in motion; and, when shut, they should be perfectly tight.

140. On pipes of small diameter a conical stop-cock may be used. It is represented by Figure 2. Plate II. A cone $h\,h$, in which is an opening of exactly the same diameter as the pipe, is turned by means of a lever p in the barrel b. By means of the wedge q it is pressed against the plate g, and thus against the barrel, as tight as may be required. In very large pipes it has been found that cocks of this description are difficult to work, and liable to break.

141. Figures 3. and 4. of Plate II. represent a very good kind of stop-cock. It consists of a plate, which is made to slide up and down by means of a vertical screw. This screw and plate are in two pieces, so that a slight horizontal motion may take place in the latter, and the water presses it tightly against the end of the pipe. This answers the purpose perfectly well, and is not very expensive.

142. The following Table of the weight of cast iron pipes is taken from "Adcock's Engineer's Pocket-Book, for the year 1833."

TABLE.

Weight of Cast Iron Pipes.

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.		
Inch.	In.	Ft. In.	C.	qr.	lb.	Inch.	In.	F. In.	C.	qr.	lb.
1		3.6	0	0	12	6		9	2	0	0
		3.6	0	0	21			9	2	2	21
1½		4.6	0	0	21			9	3	1	17
		4.6	0	1	4			9	4	0	16
2		6.0	0	1	8		1	9	5	2	20
		6	0	2	0	6½		9	2	0	16
2½		6	0	1	16			9	2	3	20
		6	0	2	10			9	3	2	21
		6	0	3	10			9	4	1	21
3		9	0	2	20		1	9	6	0	14
		9	1	0	6	7		9	2	1	7
		9	1	1	12			9	3	0	7
		9	1	3	6			9	3	3	20
		9	2	1	0			9	4	3	5
3½		9	0	3	0		1	9	6	2	4
		9	1	0	21	7½		9	2	2	4
		9	1	2	14			9	3	1	6
		9	2	0	8			9	4	0	22
		9	2	2	0			9	5	0	10
4		9	1	1	10		1	9	7	0	0
		9	1	3	12	8		9	3	2	4
		9	2	1	12			9	4	1	25
		9	2	3	21			9	5	1	18
4½		9	1	2	2		1	9	7	1	16
		9	2	0	4	8½		9	3	3	2
		9	2	2	14			9	4	2	26
		9	3	0	21			9	5	2	22
5		9	1	2	22		1	9	7	3	8
		9	2	1	10	9		9	4	0	0
		9	2	3	17			9	5	0	4
		9	3	1	24			9	6	0	2
5½		9	1	3	10		1	9	8	0	26
		9	2	2	0	9½		9	4	0	18
		9	2	3	18			9	5	1	0
		9	3	3	7			9	6	1	6
	1	9	5	0	12		1	9	8	2	20

TABLE — *continued.*

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.		
Inch.	In.	Ft. In.	C.	qr.	lb.	Inch.	In.	Ft. In.	C.	qr.	lb.
10	½	9	4	1	10			9	7	1	12
		9	5	1	26			9	8	3	16
		9	6	2	14		1	9	11	3	24
	1	9	9	0	8	14	½	9	6	0	4
10½	½	9	4	2	14			9	7	2	16
		9	5	3	7			9	9	1	0
		9	7	0	0		1	9	12	1	14
	1	9	9	2	0	14½	½	9	6	0	24
11	½	9	4	3	14			9	7	3	14
		9	6	0	11			9	9	2	2
		9	7	1	7		1	9	12	3	6
	1	9	9	3	20	15	½	9	6	1	21
11½	½	9	5	0	7			9	8	0	14
		9	6	1	12			9	9	3	7
		9	7	2	8		1	9	13	0	26
	1	9	10	1	2		1¼	9	16	3	5
12	½	9	5	0	24	15½	½	9	6	2	14
		9	6	2	8			9	8	1	4
		9	7	3	20			9	10	0	10
	1	9	10	3	0		1	9	13	2	17
12½	½	9	5	1	16		1¼	9	17	1	6
		9	6	3	9	16	½	9	7	0	22
		9	8	1	0			9	8	3	7
	1	9	11	0	21			9	10	1	20
13	½	9	5	2	20		1	9	14	0	8
		9	7	0	14		1¼	9	17	3	14
		9	8	2	7		1½	9	21	3	4
	1	9	11	2	12		2	9	29	3	21
13½	½	9	5	3	7						

CHAPTER VII.

OF PUMPS.

143. When there is in the immediate vicinity of a city a river or lake, whose waters need only to be raised to such a height as to procure a sufficient head for their distribution in the most elevated quarters of the city, pumps may be used with great advantage. Whether the motive power be a steam-engine, or a water-wheel, or any other which circumstances may render preferable, the construction of the pumps is essentially the same. There are two kinds of pumps, the *lifting* and the *forcing* pump, which we will now proceed to describe.

144. The lifting pump (Figure 11. Plate I.) is composed of a pipe P, in which plays a piston furnished with a valve opening upwards. This pipe is joined at its lower extremity to a suction-pipe S, as it is called, the end of which is plunged into the water. There is at the place in which the two pipes unite, and which should be very near to the point where the piston stands when at its lowest position, another valve also opening upwards. The manner in which the pump acts is very simple. Suppose in the first place

the piston at its lowest position *L* and both valves shut. The water in the surrounding reservoir is acted upon by the atmospheric pressure, and that which is inside the suction-pipe is acted upon by the elasticity of the air contained in *S*; so that, the pressure being equal before the pump is put into operation, it will stand at the same level in the pipe and in the reservoir. The piston is now raised to *L'*. A vacuum is thus formed between the fixed valve and the piston, and the air in *S* forces the valve open and rises till it fills the whole space between *L'* and the surface *R* of the reservoir. This air now evidently occupies a greater space than before; therefore its elasticity, and consequently its pressure upon the water in the suction-pipe are less. But the outward atmospheric pressure upon *R* remains the same. Therefore the water must rise in the suction-pipe until the weight of the column raised, added to the elasticity of the rarefied air above it, may just balance the atmospheric pressure. The piston is now pushed down. As it descends, the lower valve is shut, and the air which is above it is forced into a less and less space, and therefore its elasticity will become continually greater and greater until it is sufficient to open the piston-valve and escape.

On raising the piston a second time, the same effect is produced as before. The air above the water in the suction-pipe is rarefied still more; its elasticity is diminished, and the water rises higher. After a greater or less number of strokes, according to the length of the suction-pipe and the play of the piston, the water will rise above the fixed valve. The piston in descending will then press the water, which will force its way through the piston-valve and rest upon it. The next time the piston rises it will lift the column of water above it, and continue thus at each stroke to raise a column which increases in height, until it reaches the outlet at the top, or until the force which moves the piston is no longer able to overcome the weight of water and the friction of the parts.

145. The water rises in the suction-pipe in virtue of the atmospheric pressure, which, in a vacuum, would hold up a column about 34 feet in height. The height of the piston-valve, when at the highest point of the stroke, above the surface of the water in the well or reservoir must, therefore, be less than 34 feet, or the water cannot follow it. Generally speaking, it is made considerably less, in order to allow for the imperfections in the construction, as it is difficult to make the piston perfectly tight.

146. In order to estimate the force necessary to move the piston, we may remark, in the first place, that, when it descends, we have only to exert a sufficient force to overcome the friction of the parts, diminished by the weight of the piston and rod. When it rises, it is pressed down upon its upper side, by the weight of the column of water above it, which we will call W, and the atmospheric pressure represented by a column of the height H. It is pressed upwards on its under side also by the atmospheric pressure transmitted from the open surface of the reservoir through the water in the pipe, diminished by the weight of the water standing in the pipe between the piston and the reservoir. Calling this last weight w, the upward pressure against the piston is $H - w$. The effort necessary to raise the piston is the difference between the pressures on its two sides or $(H + W) - (H - w)$, equal to $W + w$; that is, the sum of the weights of the column above the piston and the column below it. To this should be added the force necessary to overcome the friction of the parts, and the weight of the piston and rod.

147. The quantity of water raised by each stroke, after the pump is in full operation, is equal to the area of the piston, multiplied by the length of stroke. This supposes, however, that a perfect vacuum is formed, and that there is no leakage. Generally, from imperfections in

the machinery, a less quantity is raised, which seldom surpasses the $\frac{4}{5}$ of this theoretical quantity.

148. It may be remarked, that, if the piston at the lowest point of its stroke does not reach the fixed valve, but stops at a level L'' for instance, it is possible that the pump may fail to produce its effect. Suppose for instance after one or two strokes, the piston to be at L', the water having risen a certain height h in the suction-pipe; the air in both pipes has the same density or elastic force, the lower valve is therefore equally pressed on both sides, and is shut by its own weight; the piston-valve is shut by the atmospheric pressure. The piston is now pushed down to L''; but, if the rarity of the air be such that, when after occupying the space LL' it is forced into the space LL'', its density is still less than that of the atmosphere, it will not open the piston-valve. When the piston is next raised and reaches L', the air resumes the elastic force it had before the last stroke, and the fixed valve, being equally pressed on both sides, also remains shut. Thus we see we may move the piston as long as we please, and no effect will be produced.

The height LL'', sufficient to prevent the operation of the pump, may be easily calculated. We have seen that the elasticity of the air beneath the piston, is equal to the atmospheric

pressure due to a height H, minus the height h of the column raised in the pump between the piston and the surface of the reservoir, or is measured by a height $H - h$. Calling a the area of the piston, and l the distance between the fixed valve and the highest point of the stroke, the volume of the air in the pipe P, beneath the piston when at L', will be la. Calling l' the stroke of the piston, the volume of this air when condensed will be $(l - l')\,a$. Its elastic force will be inversely as its volume, or as $H - h$ multiplied by the ratio of la to $(l - l')\,a$. In order that it may raise the piston-valve, it is necessary that this force should exceed the atmospheric pressure, represented by H, or that we should have

$$\frac{(H - h)\,la}{(l - l')\,a} > H,$$

whence we deduce

$$lh < l'H, \quad \text{or} \quad \frac{h}{H} < \frac{l'}{l}.$$

As soon as the condition expressed in this formula ceases to exist, or whenever the length of stroke divided by the greatest height of the piston above the fixed valve, is less than the quotient of the height to which the lower column is raised, divided by the height of the atmospheric column, the pump will cease to work. Hence we see the necessity of making the pis-

ton descend as nearly as possible to the fixed valve.

149. The forcing pump (Fig. 12. Plate I.) is composed of a suction-pipe S, connected with another pipe P in which plays a solid piston. At the junction of the two pipes, there is a valve V opening upward as in the lifting pump. Connected with the upper one of these two pipes is a third, near the entrance of which is a valve V' also opening upwards.

When the piston rises, the water is raised in the suction-pipe by the excess of atmospheric pressure on the surrounding surface of the reservoir, exactly as in the lifting pump. When the piston descends, the air cannot pass the piston nor escape by the lower valve, but it forces open the valve V' and escapes up the pipe P'. After a sufficient number of strokes, the water rises and fills the space between the piston and the valve V. The piston then descending forces the water through the valve V' into the pipe P', and the same valve shutting as soon as the piston rises again, prevents the water from flowing back. The succeeding stroke throws an additional quantity into the pipe, and the column thus raised will continue to increase in height until it reaches the outlet at the top of the pipe P'.

150. The remarks in Article 145 with regard to the height to which we may raise the piston

above the surface R, apply to this case also; and, from the considerations offered in Article 148, we see that it will evidently be proper to place the opening of the pipe P' as near as possible to the fixed valve of the suction-pipe.

151. In order to estimate the force necessary to keep this pump in operation, we may remark in the first place that when we attempt to raise the piston, the pressure on its upper surface is the atmospheric pressure represented by H, and that on its under side is the pressure H minus the weight of the column of water h contained between it and the surface of the reservoir. The difference h of these two pressures, or the weight of this column, added to the weight of the piston and rod, and the friction, is the measure of the force necessary to raise the piston. When we push down the piston, on the other hand, the pipes P and P' being both open at top, the atmospheric pressure is the same on both sides of the piston, and therefore may be left out of account; and the resistance to be overcome, or the measure of the force requisite to move the piston, is evidently the weight of the column of water in P' above the level of the piston. To this we must add the friction of the parts, diminished by the weight of the piston and rod. Thus we see a difference in the force necessary to work the two kinds of pump we have been con-

sidering ; inasmuch as in the first all the effort is exerted in one direction, namely in raising the piston, while in the second it requires a force alternately upward and downward.

152. As in all machines of this nature there is an advantage in applying as far as possible the same amount of power uniformly, we may remark, that, if the water is to be raised by a pump of the latter description to a height above the reservoir less than twice the height due to atmospheric pressure, we can make such an arrangement that the same force will be necessary to move the piston in either direction. For, if the valve V be placed at such a height, that the middle of the stroke of the piston shall be half way between the surface of the reservoir and the level at which the water is delivered, there will evidently be the same weight of water to overcome as it moves either up or down.

153. We see here an advantage in the forcing pump above the other, in our being able to raise water to a given height with a less force. For if, with a lifting pump, we are to raise water 50 feet for instance, we shall be obliged at each stroke, according to Article 146, to raise a column of 50 feet in height. But with a forcing pump, supposing a 4 feet stroke, the greatest force necessary to use, either in raising or lowering the piston, need be only sufficient to raise a

column of 27 feet. We suppose the friction to be the same in both cases, and therefore leave it out of account in the comparison. The weight of the piston being small, we have neglected it.

154. In order to render the motion of the water in the pipe P' more regular, it is customary to connect with it an air vessel M. When the water is forced into this pipe, the air in M is condensed by the pressure, and when the valve shuts again while the piston is rising, the condensed air reacts by its elasticity upon the water, and forces the stream to continue to flow.

155. The moving force to be employed to raise water in pumps will vary according to local circumstances. If the stream of water whence the supply is taken be of sufficient force, the use of a water-wheel would seem the most simple and economical. Windmills have sometimes been used, especially in Holland, but can only answer the purpose of an occasional supply, on account of their irregularities and interruptions. Steam is perhaps the agent most universally applied at the present day, where large quantities are to be raised constantly. Of course this is not the place to enlarge upon the construction of this kind of machinery, for which particular treatises upon steam or water power should be consulted. Yet it seems proper to give some account of the comparative results obtained in

different instances by these various machines, in order that we may possess some data whereby to determine their relative expense and expediency.

156. An experiment tried by Mr. Anderson, in order to estimate the work of the steam-engines used at the Chelsea Water-Works, by the Grand Junction Company, gave the following results. *

In 12 hours, the water in one of the Paddington reservoirs is raised 21 inches.

The surface of the reservoir at that level is 157,400 square feet.

Thus the volume of water furnished in that time is 275,450 cubic feet.

The height to which the water is raised is 100 feet ; and it is conveyed to the reservoir by a conduit 15,240 feet in length.

The mean diameter of the conduit is 25½ inches.

The machine is one of Boulton and Watt's, and of 100 horse power. The quantity of coals consumed in 12 hours is 77 bushels.

In order to estimate the work of this engine, we may remark, that it not only raises the above quantity of water 100 feet, but it communicates to it an impulse sufficient to overcome the

* Genieys. Essai sur les Moyens de conduire, d'élever, et de distribuer les Eaux.

resistance of the pipe, and to drive the water through it with its actual velocity of 1.798 foot per second; which we can deduce from the discharge and the section of the pipe. We ought, therefore, for the whole amount of the work performed by the engine, to take not only 275,450 cubic feet raised 100 feet, but suppose this whole quantity raised, in addition to the 100 feet, such a height as would be requisite to produce in the pipe the actual velocity.

Using the formula of Article 36,

$$v = 48.53 \sqrt{D \frac{\zeta}{\lambda}},$$

we have $v = 1.798$, $D = 2.125$, $\lambda = 15{,}240$;

hence $\zeta = \frac{v^2 \lambda}{(48.53)^2 D} = \frac{(1.798)^2\, 15240}{(48.53)^2\, (2.125)} = 9.84.$

We can estimate the work of this engine therefore at 275,450 cubic feet or 275,450 × 62.5 = 17,215,624 lbs. raised 109.84 feet by 77 bushels of coals, or 24,558,000 lbs. raised one foot by one bushel.

Perhaps the best information to be obtained with respect to the work performed by large engines, is that contained in the reports of Messrs. Thomas and John Lean. * The proprietors of

* Nicholson's Operative Mechanic. Rees's Cyclopædia, art. *Steam-Engine.*

the different mines in Cornwall, being much interested in determining the comparative excellence of the engines there used for raising water, appointed them to make a monthly report on the performance of the various engines, giving them all possible facilities for doing it accurately.

These reports were begun in August, 1811.

In December, 1812, the average performance of 12 engines was 17,075,000 lbs. raised one foot high by one bushel of coals.

In December, 1812, the average performance of 19 engines was 18,200,000 lbs.

In December, 1813, the average performance of 29 engines was 20,162,000 lbs.

In December, 1814, the average performance of 29 engines was 19,784,276 lbs.

In March, 1815, the average performance of 34 engines was 20,766,820 lbs.

In 1816 the three best engines were

(1.) Stray Park. A 63 inch cylinder, 7 feet 9 inches stroke, single acting. The pressure on a square inch of piston 9 lbs. Its performance in four different months was 31,000,000 lbs., 31,250,000 lbs., 28,000,000 lbs., and 28,500,000 lbs., raised one foot by each bushel of coals.

(2.) Wheal Abraham mine. A single engine, 63 inch cylinder, working an 8 feet 3 inches stroke, under a pressure of 9.4 lbs. per square

inch of piston. Its produce was 22,000,000 lbs., 29,250,000 lbs., and 32,000,000 lbs., raised one foot by a bushel of coals.

Another month, when the same engine was working with 7.9 lbs. on a square inch, its produce was 28,318,860 lbs.

(3.) Oatfield new engine. A 70 inch cylinder, 8 feet 6 inches stroke, 9.9 lbs. pressure. The performance in four different months was 22,500,000 lbs., 26,500,000 lbs., 29,000,000 lbs., and 29,000,000 lbs., raised one foot high by a bushel of coals.

The abovementioned engines are Boulton and Watt's.

The performance of Woolf's engines is greater.

The engine at Wheal Vor has a great cylinder of 53 inches diameter, and 9 feet stroke; and the small cylinder is about one fifth of the contents of the great one. The pressure is 14.1 lbs. per square inch on the surface of the great piston, and it makes 7.6 strokes in a minute. In March, 1816, its performance was 48,432,702 lbs.; in April, 44,000,000 lbs.; in May, 49,500,000 lbs.; in June, 43,000,000 lbs., raised one foot by one bushel of coals.

The engine at Wheal Abraham mine has a great cylinder of 45 inches diameter, working with a 7 feet stroke, and 15.1 lbs. pressure. Its performance during the above four months was

50,000,000 lbs., 50,908,000 lbs., 56,917,312 lbs., and 51,500,000 lbs., raised one foot by one bushel of coals.

There are also in use at these mines several enormous double engines, with cylinders of 66 and 65 inches, and four of 63 inches. The best of these appears to be on the Williams' mine; cylinder 65 inches diameter, stroke 8 feet 9 inches under a pressure of 16.6 lbs. per square inch. It works ten pumps. Its performance in June, 1816, was 30,074,507 lbs., raised one foot high by one bushel of coals. All the other double engines are less. One of the 63 inch cylinders raises 27 millions; the others 25, 22, 21, and even 17 millions.

157. According to a Report made by M. de Prony on the two engines at the Gros Caillou in Paris, the first, which is on Watt's plan, raises 11,720,700 lbs. one foot high with one bushel of coals; and the second, which is on Woolf's plan, raises 13,410,000 lbs. The performance of a high-pressure engine which raises water at the town of Béziers, as given by M. Maffre, is 11,553,000 lbs. In the last three cases the numbers given express the *useful effect* of the engines, or the quantity of water actually raised and discharged, the friction, loss by leakage, &c. being excluded. The work of the English engines is estimated by multiplying the section of

the pumps by the height of the column of water in them, and by the length of stroke, and calling this the quantity raised at each stroke. But, owing to unavoidable imperfections in the machinery, the actual performance will always be less than this.

At the Philadelphia Water-Works, according to a report made by the Watering Committee in 1824, three millions and three quarters of gallons were raised 92 feet per day, at a daily expense of $4.00 exclusive of interest and wear of machinery. This is equivalent to 13,839,841 cubic feet raised 1 foot for one dollar.

CHAPTER VIII.

RESERVOIRS, AND PIPES FOR DISTRIBUTING WATER.

158. The reservoirs, from which the water is to be distributed to different parts of the city, should be of sufficient size to contain a full supply for a number of days. This will prevent the risk of interruption in case of accidents, and allow the water time to deposit the matters it holds in suspension. If the supply be originally taken from a river, it will be liable at some seasons of the year to be very much loaded with impurities; and it is important to have, in connection with the reservoirs, filtering beds, by means of which it may be cleansed of them, before it is introduced into the distributing pipes.

159. These beds have been constructed on a very extensive scale in England. Brande's *Journal of Science*, Vol. XXVII. (1829), contains the following account of those used by the Chelsea Water-Works Company, given on the authority of Mr. Simpson, their engineer.

"The pond which contains the filter bed is 44 feet square at top and 26 at bottom, 6 feet deep. After the pond was made water-tight, with a

drain through the pond to the well, the bottom was covered with coarse gravel, in which drains were built without any cement between the joints of the bricks; they were covered with coarse gravel and then with finer gravel, with coarse sand and finer sand, until the strata of gravel and sand were each two feet thick, both gravel and sand having been selected with great care and well washed. The reservoirs were each 32 feet square at top, 20 feet at bottom, and 4 feet deep; the low-water line of them being level with the high-water line of the filter bed. The reservoirs were worked alternately on to the filter bed, and it was regulated to filter 12,000 cubic feet of water every 24 hours, and the water was remarkably pure and limpid after it had passed the bed.

"The silt which was stopped on the bed, was regularly cleaned off with a portion of the sand every 14 days. The principle of the action depends upon the strata of filtering material being finest at the top, the interstices being more minute in the fine sand than the strata below; and the silt, as its progress is arrested (while the water passes from it), renders the interstices between the particles of sand still more minute, and the bed generally produces better water when it is pretty well covered with silt, than at any other time. The silt has never been found to penetrate into the sand more than three inches, the

greatest portion being always stopped within the top half-inch of the sand; and in cleansing the silt off, it has never been found necessary to scrape any more of the sand off with the silt than the first half-inch depth, and sometimes only half that depth was required. There are small air-pipes from the drains to prevent injury to them or the filtering materials by condensation or otherwise.

"The large filter bed at Chelsea has a surface of nearly one acre on precisely the same principle. It is of sufficient capacity to filter 500,000 cubic feet every 24 hours; it is working with the greatest success during this inclement season, and, although the water on the bed is this day covered with ice five inches thick, it does not impede the filtering process."

160. The filtering beds of the Greenock Water-Works, constructed by Mr. Robert Thom, Civil Engineer, have an advantage in the facility with which they are cleansed. They are 50 feet long, 12 wide, and 8 deep. The filtering medium is a bed of clear, fine sand about 5 feet thick; and the construction is such that the water may be let in either above or below it.

When the bed has been some time in use, with the water working downward, and begins to be so clogged with foreign matters that the discharge is considerably diminished, as must always be

the case whatever be the medium, the water is introduced underneath it; and, exerting a very considerable upward pressure, it forces its way through the bed, carrying off the foreign matters which cover it, through a discharging outlet. If the filter bed has been usually worked by letting in the water beneath it, it is cleansed in a similar manner, by changing the direction and admitting the water above it. In either case the sediment is carried off, and the filter able to work again in an hour.*

161. We have seen (Chapter II.) that the formula $Q = c\sqrt{\frac{H + \zeta - H'}{\lambda}}\, D^5$ represents the relation between the discharge, the head, the length, and the diameter of a pipe, in the case of uniform motion. But when the water is to be distributed from a reservoir into the premises of individuals, the conditions of the problem are evidently no longer the same. We then seek to determine the motion in a system of pipes of different sizes, supplied by means of main pipes, from which they branch off in various directions. The manner of considering this question, and of putting it into an algebraical form, is taken from the work of M. Génieys.

* Annales des Ponts et Chaussées. 1831.

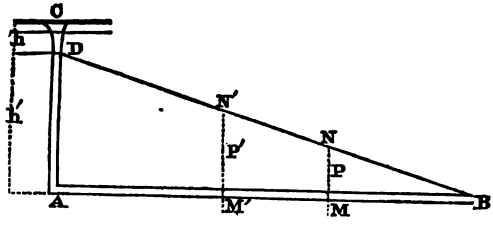
162. Let us first begin by analysing the phenomenon of the motion in a single pipe.

The water is kept in motion by the head upon the upper orifice, and the weight of that portion which is contained in the descending parts of the pipe, diminished by the portion of the head absorbed in overcoming the friction against the sides, and by the weight of the water in the ascending parts of the pipe, or above the discharging orifice.

We may consider, then, the moving force as constantly divided into three parts. The first produces the velocity of the water in the pipe; the second overcomes the friction against the sides, the third surmounts the resistance produced by the weight of that part which acts in a contrary direction to the motion.

It is this third part, which principally determines a pressure against the sides of the pipe. It is hardly any thing in rivers and open canals, where the whole moving force may be considered as employed in communicating the velocity, and in overcoming the friction against the sides; but in pipes it may be very considerable, and it is this which makes the water flow through a branch pipe, although the discharging orifice may be higher than the point of its junction with the main.

163. This will perhaps be more clearly shown by a figure. Suppose a pipe like that represent-

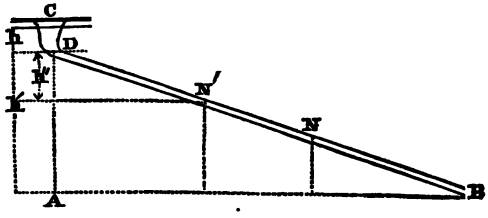


ed in CAB, AB being horizontal and AC vertical. The water in AB has of itself no tendency to move; but when pressed by the column AC, it moves with a certain velocity v due to a height h. If this water met with no resistance whatsoever in its passage, it is evident that it would flow with a velocity V due to the height $H = AC$. The fact that it does not, shows that there is a resistance, and the height $h' = H - h$ represents the column necessary to overcome this resistance. We may then, if we please, regard the pipe DAB as a machine; the friction is a force acting in the direction BA, and the weight of the column h' is a force acting in the direction AB, and these forces exactly balance each other. The column h acts moreover at one end of the machine, and transmits by means of it the pressure which produces the velocity v at the extremity B.

The impelling force represented by h' being now in equilibrium with the resistance or friction, we can easily ascertain the pressure in any part of the pipe AB. Take a section at M. The force acting from B to A is the friction on the length MB, which we shall call $F(MB)$. The pressure P, acting in a contrary direction, to be in equilibrium must be equal to it. Hence at M we have $P = F(MB)$. At M', the force acting from B to A is the friction on $M'B$ or $F(M'B)$. The pressure being P' is $P' = F(M'B)$ and so on. Now, the resistance which we are considering being proportional to the length of the pipe, or to the distance from the extremity B', and being moreover evidently nothing at B and equal to h' at A *; if we draw a straight line from D to B, it is evident that the ordinates MN, $M'N'$ will represent for any point M, M' &c. the height of the column necessary to overcome the friction between that point and the extremity, or the pressure at the point in question. Thus if at M or M' we make an opening in the pipe AB and erect a vertical pipe, the water must rise in it to the height MN, or $M'N'$.

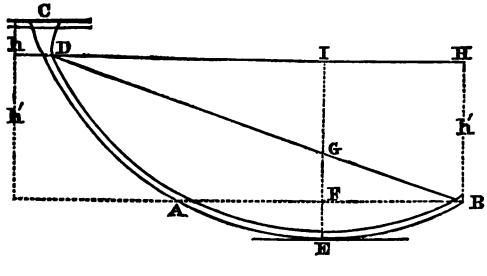
* We neglect here the small friction in the vertical part AC.

164. It will be readily admitted that if we substitute for *DAB* a regularly inclined pipe *DB*,



there will be no pressure in this pipe. For every foot of its length has a fall just sufficient to overcome the friction for that length; and we shall not alter the motion by increasing the length indefinitely. At any section N' of this new pipe, the water contained between N' and B does not exert any effort from B towards D, and the whole of the fall h'' from D to N' is employed in overcoming the friction, and there is none left to produce a pressure. If we insert a pipe upon *DB*, the water therefore will not rise in it.

165. Let us now take a third pipe *DAEB*,



in which there is an ascending part, as is much the most common case in practice.

We have here, as before, CD or h employed to produce the velocity at B, and the rest of the impelling force h' is employed to overcome the resistance.

We will suppose, as will almost always be the case in practice, that any lengths measured on the axis of the pipe, are proportional to the corresponding lengths measured horizontally between one extremity and the other; or in other words, that it will take about as much pipe to lay a mile in one part of the line as in another. This being the case, the height required to overcome the friction from D to B being h', that required to overcome the friction from D to any intermediate point E, being to h' in the proportion of ED to BD or of HD to ID, is evidently equal to IG. Thus at any point of the pipe DEB raising a vertical line, that part of it included between the horizontal DH and the line DB, will be the head required to overcome the friction from the upper extremity to the point in question; and the part included between the inclined line DB and the pipe, will represent the pressure, or the height to which the water would rise in a vertical tube. Of this, the part GF overcomes the remainder of the friction from E to B, and FE the weight of the water which

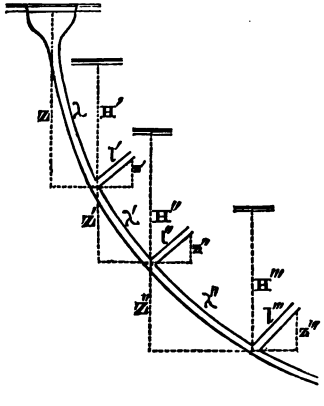
tends to flow back in the same part *EB* of the pipe.

166. In the case of a single pipe, neglecting the very small height requisite to produce the velocity, we have supposed that the height of the column which would measure the pressure, was equal to the head upon the upper extremity. It would seem natural enough to make the same hypothesis in the case of branch pipes, and to suppose that the pressure at their origin is the same as that in the main pipe at the same point, or has for its measure the height to which the water in the main pipe would rise in a vertical tube. If this hypothesis is admissible, the phenomenon of the motion may be determined.

167. In order to ascertain this important point, M. Génieys, M. Bélanger, and M. Mallet commenced a series of experiments; and the result was, that the pressure at the extremity of the branch pipe was less than that in the main, by a certain quantity represented by a height equal to three times that due to the velocity of the water in the branch. Thus the velocity in the branch pipe being 0.847 metre per second, due to a height 0.036569 metre, the pressure at its origin was less by 0.12, or nearly 3×0.036569, than in the main. Again the velocity in the branch pipe being 1.003 metre, which is due to a height of 0.05128 metre, the difference of pressure was 0.153, or nearly 3×0.05128.

The same experiments, having been repeated with various velocities, gave constantly the same results ; and it was thence concluded that for velocities between 0.20 metre and 1 metre or between 0.66 foot and 3.28 feet, the pressure in the main pipe may, in our calculations, be taken for that at the orifice of the branch, at the place of its insertion, provided that we finally add to the necessary head of water in the reservoir which produces the motion in the first instance, three times the height due to the velocity in the branch, or the quantity lost at the passage from a main to a branch.

168. This being premised, we shall now proceed to find formulæ for the motion of water in a system of main and branch pipes.



Let Q be the quantity which the main pipe is to discharge per second, in its upper portion, or above the first branch.

D the diameter of the main ;

L its length.

$\lambda, \lambda', \lambda'', \lambda''' \ldots \lambda^{n-1}$ the length

of pipe between two successive branches; so that $L = \lambda + \lambda' + \lambda'' + \lambda''' \ldots + \lambda^{n-1}$.

Z the difference of level between the water in the reservoir and the beginning of the first branch.

Z', Z'', Z''' Z^{n-1}, the differences of level between the successive branches.

H', H'', H''' H^n the height of the column of water representing the *head* at the origin of each branch, or the pressure against the sides of the main pipe.

q', d', l', z', ...
q'', d'', l'', z'', ...
q''', d''', l''', z''', ...
.
q^n, d^n, l^n, z^n, ...

} for each branch pipe the corresponding elements to Q, D, L, Z, in the main.

c = a constant coefficient.

Considering in succession each part of the pipe, we shall have

$$Q = c \sqrt{\frac{Z - H'}{\lambda} D^5} \quad \ldots\ldots\ldots\ldots (1);$$

$$q' = c \sqrt{\frac{H' - z'}{l'} d^5} \quad \ldots\ldots\ldots\ldots (2);$$

$$Q - q' = c \sqrt{\frac{H' + Z' - H''}{\lambda'} D^5} \quad \ldots\ldots (3);$$

$$q'' = c \sqrt{\frac{H'' - z''}{l''} d''^5} \quad \ldots\ldots\ldots\ldots (4);$$

$$Q - q' - q'' = c\sqrt{\frac{H'' + Z'' - H'''}{\lambda''}}\, D^5 \ldots (5);$$

.

$$q^n = c\sqrt{\frac{H^n - z^n}{l^n}}\, d^{n5} \ldots\ldots\ldots (2n);$$

making in all $2n$ equations, in which the indeterminate quantities are

$$D\ d'\ d''\ d'''\ \ldots\ \ldots\ d^n,$$
$$H'\ H''\ H'''\ \ldots\ \ldots\ H^n.$$

But we can easily eliminate the quantities H', H'' H''' . . . H^n. For, by combining successively the equations (1) and (3), (1), (3) and (5) &c., we shall have

$$Q^2\lambda + (Q - q')^2\lambda' = c^2 D^5 (Z + Z' - H'') \ \ . \ . \ (A);$$

$$Q^2\lambda + (Q - q')^2\lambda' + (Q - q' - q'')^2\lambda'' = c^2 D^5 (Z + Z' + Z'' - H''') \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (A');$$

.

$$Q^2\lambda + (Q - q')^2\lambda' \ldots + (Q - q' - q'' \ldots - q^{n-1})^2 \lambda^{n-1} = c^2 D^5 (Z + Z' \ldots + Z^{n-1} - H^n).$$

Moreover, from the equations (1) and (2), (4) and (A), (6) and (A') &c., we shall obtain

$$Q^2\lambda\, d'^5 + q'^2 l'\, D^5 = c^2 D^5 d'^5 (Z - z') \ . \ . \ . \ . \ . \ (B);$$

$$(Q^2\lambda + (Q - q')^2\lambda')\, d''^5 + q''^2 l''\, D^5 = c^2 D^5 d''^5 (Z + Z' - z'') \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (B');$$

.

$$(Q^2\lambda + (Q - q')^2\lambda' \ldots + (Q - q' \ldots - q^{n-1})^2 \lambda^{n-1})\, d^{n5} + q^{n2} l^n D^5 = c^2 D^5 d^{n5} (Z + Z' \ldots + Z^{n-1} - z^n).$$

Thus we have at last n equations between $n + 1$ undetermined quantities D, d', d'', d''' d^n.

169. It would not be sufficient to fix upon a certain value for one of the quantities D, d', d'' . . . d^n, in order to solve the question; for we should often find imaginary expressions for the other quantities.

To make this clearer, let us find by means of the equations B, B', B'', &c. the values of d', d'', d''' . . . d^n. We have

$$d'^5 = \frac{q'^2\ l'\ D^5}{c^2\ D^5\ (Z - z') - Q^2\lambda}\ ;$$

$$d''^5 = \frac{q''^2\ l''\ D^5}{c^2\ D^5 (Z + Z' - z'') - (Q^2\lambda + (Q - q')^2\lambda')};$$

.

$$d^{n5} = \frac{q^{n2}\ l^n\ D^5}{c^2 D^5 (Z + Z' \ldots + Z^{n-1} - z^n) - (Q^2\lambda + (Q - q')^2\lambda' + \ldots}$$

$$\overline{(Q - q' \ldots - q^{n-1})^2\ \lambda^{n-1}}).$$

Now, in order that we may have values of d', d'' &c. applicable to the question, these denominators must all be positive, or we must have, in general,

$c^2 D^5 (Z + Z' \ldots . + Z^{n-1} - z^n) > Q^2\lambda + (Q - q')^2\lambda' + \ldots (Q - q' \ldots . - q^{n-1})^2 \lambda^{n-1}$.

This condition limits very much the number of values which we may give to D.

170. We have hitherto supposed that the level of the reservoir which produces the head, is constant. When the reservoir is filled by means of a canal or pipes bringing water from a distance by means of a sufficient fall, this will

generally be the case. But if the water is pumped up into the reservoir, the level may be variable, and another indeterminate quantity is introduced into the problem. In this case the n equations (B) (B') (B'') (B^n) do not change, but they contain $n + 2$ indeterminate quantities,

$$D,\ d',\ d'',\ d''' \ldots . d^n \text{ and } Z.$$

171. If we wish to know the values of H', H'', H''' . . . H^n, which give the head or pressure at the origin of each branch, we shall find them by means of the equations (1) (A) (A') . . &c. From these we deduce

$$H' = Z - \frac{q^2 \lambda}{c^2 D^5};$$

$$H'' = Z + Z' - \frac{Q^2 \lambda + (Q - q')^2 \lambda'}{c^2 D^5};$$

$$H''' = Z + Z' + Z'' - \frac{(Q^2 \lambda + (Q - q')^2 \lambda' + (Q - q' - q'')^2 \lambda'')}{c^2 D^5};$$

.

$$H^n = Z + Z' + Z'' \ldots . + Z^{n-1} - \frac{Q^2 \lambda + (Q - q')^2 \lambda' \ldots + (Q - q' \ldots - q^{n-1})^2 \lambda^{n-1}}{c^2 D^5}.$$

These equations show, that, when Q, q', q'' . . . q^{n-1}, λ, λ', λ'' . . . Z, Z', Z'' . . . remain the same, H', H'', H''' and D augment and diminish simultaneously.

172. Finally, v being the velocity of the water in a pipe whose diameter is D, and Q the discharge per second, g the velocity which gravity imparts at the end of a second, h being the height due to the velocity v, and $\pi = 3.1415926$, we have

$$v = \frac{4\,Q}{\pi\,D^2} = 1.2732\,\frac{Q}{D^2};$$

$$h = \frac{v^2}{2g} = 0.015537\,v^2.$$

By means of these equations we can calculate the velocity and the height to which it is due, when the diameter has been determined.

173. We will now show by an example the manner of applying the preceding formulæ.

We will suppose a main pipe leading from a reservoir, and passing through the centre of the quarter of the city which it is to supply. With this pipe are connected ten branches, which run in various directions, passing through the adjoining streets; and from these branch pipes, smaller pipes convey the water into the private dwellings. The quantity of water to be furnished is 20,000 cubic feet in 24 hours, or 124,642 gallons. We will suppose moreover, for greater security and for more simplicity in our calculations, that all the water which enters a branch pipe is discharged at its extremity; for it is evident, that, if the diameter and direction of the pipe are so determined that this condition may be fulfilled, they

will certainly answer for the case where the water is successively drawn from the pipe at various points of its length.

We will call moreover the point, at which these branch pipes discharge, 20 feet above a certain assumed base; and suppose that the origin of the first branch is 10 feet above the same base. The question is, what must be the height of the reservoir, and what the diameter of the several pipes.

174. In Table I. we have arranged in order the several data of the question, and from it we can easily calculate Table II.

TABLE I.

Main Pipe.	Lengths intercepted or values of λ, λ', λ'', . . . between the successive branches.	Branch Pipes.	Lengths $l_{\rm I}$, $l_{\rm II}$, $l_{\rm III}$, $l_{\rm IV}$, ...	Discharge in 24 hours.
	feet.		feet.	cub. feet.
Reservoir to Branch I	300	I	$560 = l^{\rm I}$	1780
Branch I to II	150	II	$1200 = l^{\rm II}$	3370
II to III	260	III	$670 = l^{\rm III}$	1300
III to IV	340	IV	$890 = l^{\rm IV}$	1560
IV to V	525	V	$900 = l^{\rm V}$	2500
V to VI	600	VI	$200 = l^{\rm VI}$	600
VI to VII	230	VII	$678 = l^{\rm VII}$	1790
VII to VIII	210	VIII	$940 = l^{\rm VIII}$	2400
VIII to IX	497	IX	$1100 = l^{\rm IX}$	2900
IX to X	360	X	$700 = l^{\rm X}$	1800
	$3472 = L$		7838	20000

TABLE II.

No. of branch	Discharge in 24 hours.	Discharge per second.	$Q, Q-q^{I}, Q-q^{I}-q^{II}, Q-q^{I}-q^{II}-q^{III}$, &c.	$Q^2\lambda, (Q-q^{I})^2\lambda^{I}, (Q-q^{I}-q^{II})^2\lambda^{II}$, &c.	$Q^2\lambda, Q^2\lambda+(Q-q^{I})^2\lambda^{I}, Q^2\lambda+(Q-q^{I})^2\lambda^{I}+$ &c.
	cub. feet.				
I	1780	$0.020602 = q^{I}$	0.231481	16.075	16.075
II	3370	$0.039005 = q^{II}$	0.210879	6.670	22.745
III	1300	$0.015046 = q^{III}$	0.171874	7.680	30.425
IV	1560	$0.018056 = q^{IV}$	0.156828	8.362	38.787
V	2500	$0.028935 = q^{V}$	0.138772	10.110	48 897
VI	600	$0.006944 = q^{VI}$	0.109837	7.239	56.136
VII	1790	$0.020718 = q^{VII}$	0.102893	2.435	58.571
VIII	2400	$0.027777 = q^{VIII}$	0.082175	1.418	59.989
IX	2900	$0.033565 = q^{IX}$	0.054398	1.471	61.460
X	1800	$0.020833 = q^{X}$	0.020833	0.156	61.616
	20000	0.231481			

175. The general equation which gives the value of the diameter is (Art. 169)

$$d^{n5} = \frac{q^{n2}\, ln\, D5}{C^2\, D5\,(Z + Z' + Z'' + \ldots + Z^{n-1} - z^n) - (Q^2\lambda + (Q-q^{I})^2\lambda_{I} \ldots + Q - q^{I} - q^{II} \ldots - q^{n-1})^2\lambda^{n-1}}.$$

Now $Z' + Z'' + Z''' \ldots + Z^{n-1} - z^n$, as may be seen by the Figure, is evidently equal to the difference of level between the origin of the first branch, and the point at which the branches discharge; or is equal to $10 - 20 = -10$. Moreover, in order to be on the safe side, we shall take for c its least value given in Article 39, or $c = 33.094$.

Substituting these values in the above equation, it becomes

$$d_n{}^5 = \frac{q^{n\,2}\, l^n\, D^5}{(33.094)^2\, D^5\,(Z-10) - (Q^2\,\lambda + (Q-q')^2\,\lambda' + \ldots +}$$

$$(Q-q'-q''\ldots-q^{n-1})^2\,\lambda^{n-1}.$$

We can now obtain the diameter of any branch pipe, No. X. for instance, by substituting in this equation for q^x, l^x, $Q^2\,\lambda + (Q-q')^2\,\lambda' + \ldots + (Q-q'-q''\ldots.-q^{IX})^2\,\lambda^{IX}$, their values taken from Tables I. and II. Thus we shall have

$$d^{x5} = \frac{(0.020833)^2\,700\,D^5}{(33.094)^2\,D^5\,(Z-10) - 61.616} \qquad (C).$$

176. In this equation we may fix upon any value for D or for Z, provided that when one of them is determined, the other shall be such as to render the denominator positive. We will suppose various values for D, and see what corresponding values of Z would make the denominator equal to zero.

We shall find *

$D =$	$0.20^{ft.}$	0.30	0.40	0.50	0.60
$Z =$	185.810	33.152	15.494	11.800	10.723.

We see by this Table that the requisite head increases in a very rapid ratio as we diminish D. Thus if $D = 0.20$, Z being 185.810, the level

* $(33.094)^2\,D^5\,(Z-10) = 61.616$;

$$Z = \frac{61.616}{(33.094)^2\,D^5} + 10.$$

of the surface of the reservoir must be something more than 195.810 above the assumed base. And from such an excessive height, independently of the difficulty of placing a reservoir there, the velocity in the branch pipes would become enormous, and their diameters would consequently have to be smaller than would be convenient in practice.

Let us suppose then $D = 0.40$. Then, in order that the denominator of the formula may remain positive, it is only necessary to give to Z a value a little greater than 15.494. We will call it $Z = 16$, in which case the surface of the water in the reservoir will stand at $16 + 10 = 26$ feet above our base.

177. If we now substitute in the formula (C) the values 0.40 and 16 for D and Z respectively, we shall find the diameter d^{x};

$$d^{x5} = \frac{(0.020833)^2\ 700\ (0.40)^5}{(33.094)^2\ (0.40)^5\ 6 - 61.616} = \frac{0.003111}{5.674};$$

$d^{x} = 0.223$ feet.

We can now find from a similar equation the values of d^{ix}, d^{viii}, d^{vii}, . . . &c., retaining always for D and Z the same values as above.

$$d^{ix5} = \frac{(0.033565)^2\ 1100\ (0.40)^5}{(33.094)^2\ (0.40)^5\ 6 - 61.460} = \frac{0.012690}{5.83};$$

$d^{ix} = 0.293$.

The following Table contains the results of these calculations, and also the velocities in the

main and branch pipes, calculated by the formula $v = 1.2732 \frac{Q}{D^2}$ (Art. 172.)

TABLE III.

Branches.	Diameters.	Velocity in the branches.	Velocity in parts of the main pipe.	
	feet.			
I	$d^{I} = 0.137$	1.406	1.842	The velocity in the main pipe diminishes constantly, since it conveys less and less water after passing each branch pipe successively. The diameter is 0.40.
II	$d^{II} = 0.211$	1.114	1.678	
III	$d^{III} = 0.136$	1.042	1.367	
IV	$d^{IV} = 0.160$	0.900	1.248	
V	$d^{V} = 0.211$	0.826	1.104	
VI	$d^{VI} = 0.098$	0.928	0.874	
VII	$d^{VII} = 0.203$	0.642	0.819	
VIII	$d^{VIII} = 0.252$	0.557	0.654	
IX	$d^{IX} = 0.293$	0.496	0.433	
X	$d^{X} = 0.223$	0.535	0.166	

178. We have now determined the proper height of the surface of the water in the reservoir, and the diameter of the main and secondary pipes. But in our calculations we have regarded the head or pressure upon the orifice of the branch pipe, as equal to that in the main, at the part adjoining. We must therefore make the correction spoken of in Article 167, and add to the height of the reservoir, for each change of direction, or passage from one pipe to another, three times the height due to the velocity. We should not be satisfied, however, with merely giving to our pipes the dimensions determined by

the foregoing methods, but should make them somewhat larger. Yet by means of the last Table, we can get a pretty correct idea of their comparative size, which will enable us to distribute whatever extra dimensions and strength we are willing to allow, in the most judicious manner.

With regard to air-cocks and stop-cocks, what we have said in Chapter VI. will be sufficient, without adding any thing more upon the subject here.

CHAPTER IX.

PIPES OF DIFFERENT DIAMETERS, AND JETS D'EAU.

179. If we have a pipe composed of several parts of different diameters, we can easily deduce the law of the motion of the water flowing through it from the formula of Article 35.

Let there be, for instance, two parts. Let v', D', ζ', λ', be the velocity, diameter, fall, and length belonging to the first part, and v'', D'', ζ'', λ'' the corresponding elements for the second part. We have

$$a v' + b v'^2 = \tfrac{1}{4} D' \frac{\zeta'}{\lambda'},$$

and

$$a v'' + b v''^2 = \tfrac{1}{4} D'' \frac{\zeta''}{\lambda''}.$$

The whole head, or difference of level between the extremities of the whole conduit, is

$$\zeta' + \zeta'' = \zeta.$$

From the first equation above, we deduce

$$\zeta' = \frac{4\lambda'}{D'} (a v' + b v'^2).$$

From the second we deduce

$$\zeta'' = \frac{4\lambda''}{D''} (a v'' + b v''^2).$$

If the first pipe had been used, we should have had

$$D = 1,\ \lambda = 10{,}000,\ \zeta = 20,\ j = \frac{\zeta}{\lambda} = 0.002,$$

$$\tfrac{1}{4} Dj = \frac{0.002}{4} = 0.0005000.$$

The Table of Article 44 shows us that this corresponds to $v = 2.09$, which would therefore be the velocity. Hence

$$Q = \frac{\pi}{4} \times 2.09 = 1.6415.$$

If the second pipe had been used, we should have had

$$D = 0.83,\ \lambda = 10{,}000,\ \zeta = 20,\ j = 0.002,$$

$$\tfrac{1}{4} Dj = 0.83 \times 0.0005 = 0.0004150;$$

and the Table gives us for the velocity $v = 1.90$. Hence

$$Q = \frac{\pi}{4} (0.83)^2 \times 1.90 = 1.0280.$$

If the third pipe had been used, we should have had

$$D = 0.75,\ \lambda = 10{,}000,\ \zeta = 20,\ j = 0.002,$$

$$\tfrac{1}{4} Dj = 0.75 \times 0.0005 = 0.0003750;$$

and the Table gives $v = 1.80$.

Hence

$$Q = \frac{\pi}{4} (0.75)^2 \times 1.80 = 0.7952.$$

183. We thus see, that a pipe consisting of parts of different diameters, supposing them of considerable length, will discharge more than a continued pipe of the smallest diameter; and this seems a reasonable supposition *à priori*, since in the part where the diameter is greater, there is less friction, and consequently less resistance to the motion, than if the small pipe were continued. Yet it is true, according to the remark in Article 46, that a number of sudden enlargements, after which the section is again reduced, are a disadvantage and diminish the discharge.

184. If, in the equation of Article 180, we neglect the term containing the first power of v', which we may generally do unless the velocity be very small, we shall have

$$\zeta = 4b\left\{\frac{\lambda'}{D'} + \frac{\lambda''D'^4}{D''^5} + \frac{\lambda'''D'^4}{D'''^5} + \&c.\right\}v'^2,$$

or $\zeta = Bv'^2$; whence $v' = \sqrt{\frac{\zeta}{B}}$,

a much simpler expression.

Applying this formula to the example of Article 181, we shall have

$$v' = \sqrt{\frac{20}{13.02896}} = 1.239,$$

and

$$Q = \frac{\pi v'}{4} = 0.9731.$$

185. In situations where ornament is desirable, as well as use, the water, instead of being simply discharged into a reservoir from the midst of a horizontal conduit, is sometimes made to rise from the centre of the basin in a vertical jet. Besides the delightful freshness which it thus imparts to the surrounding air, its brilliant reflection of light and the sprightliness of its motion render it one of the most beautiful objects that can be imagined.

The conditions to be fulfilled, in order to produce a jet of a given size and height, are easily determined. The first is, that the water, as it passes the orifice, may have the velocity due to the height to which it is to rise vertically; the second, that the conduit may supply exactly the quantity which the jet discharges.

The first condition shows us what the discharge must be, the section of the jet and its velocity, deduced from the height, being both known. The second condition requires, that the velocity in the pipe should be to that at the beginning of the jet inversely as the section of the pipe is to that of the jet. Fixing therefore upon a certain diameter for the pipe, the velocity of the water in it is determined. Knowing the length of the pipe, its diameter, and the velocity, we can easily determine, by the formulæ of Article 43, what head will be required to overcome the friction, and produce this velocity. Adding this to the height to

which the water is to rise, we shall have the whole height required from the surface of the reservoir to the orifice.

186. To express the motion by a formula, let

ζ' denote the height of the jet,

D its diameter, measured at the contracted section just above the orifice,

v' its initial velocity $= \sqrt{2g\zeta'}$,

D''. . . . the diameter of the conduit,

λ'' its length,

ζ'' the head employed in overcoming the friction and producing the motion,

v''. . . . the velocity in the conduit,

ζ the whole head from the surface of the reservoir to the orifice whence the jet issues.

The initial velocity of the jet will be $\sqrt{2g\zeta'}$, which we call v'. The velocity in the conduit will be to v' in the inverse ratio of the sections; or $v'' = \frac{v'D'^2}{D''^2}$. The head required to produce this velocity is

$$\zeta'' = \frac{4\lambda''}{D''}(a v'' + b v''^2);$$

the whole height of the reservoir will therefore be

$$\zeta = \frac{4\lambda''}{D''}(a v'' + b v''^2) + \zeta'.$$

187. If there are curves or other obstructions in the pipe, they should be taken into account ac-

cording to the rules given in the last articles of Chapter II., and will increase the height which it is necessary to give to the reservoir. It should be remarked, that on account of the phenomenon of the contraction, the area of the orifice cannot be taken for the section of the jet; but the diameter of the latter being D', we must make the diameter of the orifice equal to $\frac{D'}{\sqrt{m}}$, m being the coefficient of contraction. (See Articles 58 and 59). The area will then be $\frac{\pi D'^2}{4m}$, and, multiplying this by m, we have $\frac{\pi D'^2}{4}$ for the section of the jet. The best kind of orifice is simply a hole made in a thin plate, without any additional tube. The water will issue in a direction perpendicular to the plate; hence, by curving it properly, we can vary the direction at pleasure.

188. The resistance arising from the air, which rapidly increases with the velocity of the jet, and that occasioned by the water which falls back upon itself, were not taken into account in Article 186. Hence the height will be rather less in practice, than we should deduce from the formula. According to Mariotte's experiments (*Traité du Mouvement des Eaux*), the loss of height is as the square of the height of the jet; and a jet 5 feet in height requires an additional inch in the reser-

voir to compensate for the loss. Hence a jet H feet in height requires an addition of $\frac{H^2}{25}$ inches, or $\frac{H^2}{300}$ feet.

189. To give an example of these calculations, we will suppose that at the end of a pipe 0.75 foot in diameter and 15,840 feet in length, we wish for a jet 25 feet high, whose smallest section shall be 0.1 foot. What must be the difference of level between the surface of the water at the fountain-head, and the orifice whence it issues?

The initial velocity of the jet is $v' = \sqrt{2g.25} = 40.1134$.

The velocity in the pipe is

$$v'' = \frac{40.1134 \times 0.01}{0.75 \times 0.75} = 0.713.$$

In the Table of Article 44, we find by a simple proportion, that to $v'' = 0.713$ corresponds

$$\tfrac{1}{4} D''j = 0.0000663 = \frac{D''\zeta''}{4\lambda''}.$$

Hence

$$\zeta'' = \frac{0.0000663 \times 15{,}840 \times 4}{0.75} = 5.60.$$

We have then, $\zeta = 25 + 5.60 = 30.60$.

To this we must add, according to Mariotte's rule, $\frac{25 \times 25}{300} = 2.08$; and the whole height of the reservoir above the orifice must accordingly be 32.68 feet.

CHAPTER X.

ARTESIAN WELLS.

190. It seems not improper in this place to give some account of what are called *Artesian Wells*, or subterraneous springs whose waters, when they have been reached by boring from the surface, rise and maintain a much higher level than that at which they were first found. A description of the various instruments to be used in the process of boring belongs to a special treatise on this subject, such as those of Garnier and Héricart de Thury. But some information respecting the nature and geological position of the strata, where these rising springs are to be met with, will enable us to judge of the probable success of boring, before undertaking expensive operations. What we have to say on this subject will be taken from the work of Garnier, for which the author received a prize from the *Société d'Encouragement pour l'Industrie Nationale* of France, and which was published by order of the French government in 1822.

191. The first of these wells of which any account has ever been given, were those in the

French province of Artois, from which is derived their name of *Artesian*. In the common wells of the country, it was observed that the water rose to a considerable height above the level at which it was first met with; and of late years, by means of instruments specially invented for the purpose, it has been found possible to raise it from a depth of 300 feet.

The following sections show the nature and superposition of the various beds, through which the auger passed before reaching the subterraneous bodies of water, in various localities in the neighbourhood of Calais.

(1.) In the commune of Ardres.

	metres.
Sand and pebbles	1.65
Clay and sand	3.30
Sand	8.00
Clay of a yellowish color with some sand	3.30
Quicksand	11.20
Very compact black clay	5.00
Sand and pebbles	1.65
Sand containing masses of sandstone	1.30
Grey clay, mixed with a little lime	5.00
Chalk entered to the depth of	7.00
	47.40

(2.) In the commune of Annezin.

Vegetable earth	3.70
Pebbles with a little sand	3.30
Black or grey clay, very hard	23.00
Chalk entered to the depth of	12.00
	42.00

(3.) In the commune of Choques.

	metres.
Vegetable mould	5.00
Clayey earth	1.65
Quicksand	2.20
Sand and pebbles	2.00
Clay of a light grey color	6.60
Grey clay with sand	5.00
Blue clay, hard and compact	7.00
Chalk entered to the depth of	2.50
	31.95

(4.) In the commune of Aire.

Grey quicksand	1.62
Sand mixed with some earth	5.60
Sand and pebbles	3.65
Quicksand of a greenish grey color	3.65
Quicksand of a slate color	7.10
Grey clay with sand	3.30
Clayey earth of a yellowish color, with pebbles	4.40
Black clay with sand and pyrites	5.70
Grey clay with little lumps of chalk	3.65
Bluish clay of a very homogeneous texture	3.90
Very black clay	3.30
Chalk entered to the depth of	9.00
	54.87

(5.) In the commune of Merville.

Vegetable mould	1.60
Quicksand	4.00
Grey clay	30.00
Lumps of sandstone	0.16
Green, earthy clay	9.60
Ferruginous pyrites	11.30
Black sand	2.00

	metres.
Green sand	2.20
Black earthy sand	2.90
Black quicksand	14.00
Sandy clay	2.00
Very hard grey clay with pyrites	4.50
Very compact black clay	12.00
Chalk entered to a depth of	8.00
	93.26

These sections show that the waters are all contained in the fissures of calcareous rocks, covered by horizontal beds of vegetable earth, sand, rounded pebbles, and clays of various kinds, which appear to have been deposited in tranquil waters. The wells are found in a line running about northwest and southeast, which forms the line of separation between the high grounds on the southwest, and the vast plain extending to Holland on the northeast. All those dug in this vicinity exhibit the same succession of beds, and show moreover, that, in proportion as we leave the foot of the high grounds and advance into the plain, the thickness of the superincumbent beds augments, and the chalk lies at a greater depth below the surface. As we proceed in the opposite direction towards the northwest, we find the ground rising continually, and the calcareous strata showing themselves at the surface, or covered only by a layer of vegetable earth. This region presents all the geological

characters belonging to a secondary formation composed of strata of calcareous rocks, of nearly the same texture and color.

192. From this it will appear that the sedimentary formations are first met with at the line of separation between the high grounds and the plain. On one side of this line the secondary rocks appear out of the ground; on the other they are covered by a great thickness of earths of a more recent formation. It is evident, too, that it is in the neighbourhood of this line of separation, that we should undertake boring operations; since, after piercing through the impermeable beds of clay, we ought at a moderate depth to reach the strata of chalk, and experience fully shows this to be the case.

It has been found, however, that rising springs may be also met with in the deep valleys which intersect the high grounds, whenever the water can have found passage through the fissures of the chalk. At Blengel, in the valley of the Ternoise, the auger was tried in three places very near each other. The first hole was made 50 feet deep; the second 80; and the third 110. The first gave no water; the second was abandoned, because the workmen, from their want of skill, were unable to draw out the auger, which had been broken by striking upon some flint pebbles. The third, at the depth of 100 feet, appear-

ed as little likely to be successful as the others. But upon being continued ten feet farther through a very tenacious blue earth, the lower part of which was yellowish, and approaching the nature of marl, a spring was met with which rose immediately to the surface.

The section of the strata was as follows:

	metres.
Grey clay	1.30
Grey clay of a lighter color . .	2.50
Chalk (it was in this that the boring of the first two holes was stopped) . . .	28.30
Bluish clay	3.20
	35.30

Chalk — just entered.

It is evident that the waters rose here solely because they now found a vent through the stratum of clay which retained them beneath the upper stratum of chalk.

If we undertake to bore for water in the high lands, we should choose the lowest points of the valleys which intersect them. Otherwise the distance between the level to which the water will rise and the surface of the ground may be very considerable.

193. From the sections which we have given, it is to be seen that the underground waters are found only in the chalk; and this is very easily accounted for. This stratum is covered by an impermeable bed of clay; and if the latter has a

considerable extent, it is evident that all the water underneath it is subjected to a pressure and can find no vent. Rain water, and that from the various streams of the high land, enters the fissures of the chalk where it approaches the surface, and, finding free passage there, it spreads in all directions underneath the recent formations by which it is covered. Finding no ready issue, it must remain there and become more or less stagnant, in proportion as the superincumbent beds of clay are greater in extent, and the natural issues by which it can finally reach the ground are smaller and more distant. It is evident, then, that as soon as a passage for it is pierced through the clay, it will rise with a velocity due to the pressure to which it is subjected. If it is stagnant it will rise to the same level as that at which it entered the strata; but if it is moving with a certain velocity, it will rise to a height somewhat less. In order that the water may rise, it is necessary, moreover, not only that it should be prevented from extending indefinitely in breadth, but also that there should be an impermeable stratum beneath it; and in many localities it is found that the lower parts of the chalk become compact and tight.

194. At Sheerness in England, at a depth of 350 feet there was found underneath the clay a stratum of chalk containing very pure and lim-

pid water. As soon as the clay had been pierced through, the water rose to a height of 344 feet; but it afterwards fell, and remained stationary 120 feet below the surface. The formation which covers the chalk at Sheerness is recent, and is very analogous to that in the neighbourhood of Calais. It is principally composed of sand of various colors, mixed with green earth and rolled pebbles, and of a very tough, dark-colored clay, similar to some varieties of that met with in France. The localities are exactly the same in a geological point of view.

195. M. Héricart de Thury has given the following account of the beds through which the auger passed at the Barrière de Fontainebleau near Paris, and at the Courtalin paper factory near Coulommiers; and it tends to prove, that the best and most copious springs are there found in the chalk which lies underneath recent formations analogous to those of which we have spoken.

At the Barrière de Fontainebleau the section was as follows:

	metres.
Vegetable earth, sand, and gravel . .	3.82
Calcareous marl, and limestone containing marine shells . . .	16.52
Clay of various colors belonging to the argillaceous deposits . . .	11.36
Dark grey clay with pyrites . .	0.33

	metres.
Siliceous and argillaceous sand alternating with veins of sandy clay, of a dark grey color	7.47
	39.50

Immediately beneath these beds is found the great chalk formation, whose thickness is unknown.

The water found in the clay beds, which was first collected in a well at the depth of 31.70, being insufficient, it was thought advisable to bore to a greater depth. The auger first passed through a very hard bed of dark grey clay containing pyrites, 0.33 metre in thickness. Immediately after, it slipped down out of the workmen's hands through a space of 7.47 metres, exactly as if it had fallen into a deep fissure. If it had not been held by a cord passing through a ring in the handle and round the drum of a windlass, it would probably have fallen to a greater depth; for the workmen asserted, that when they attempted to draw it up, it appeared to have entered an empty space, that it did not touch any thing solid at bottom, and that it was moved to and fro as if by a strong current of water. They had a great deal of trouble in getting it up; the water followed it and disturbed their operations; but the moment it was drawn out and the opening cleared, a stream of water rushed out

above their heads to the height of 30 feet, and with such force and abundance, that they had barely time to be drawn up from the bottom of the well where they had been working, leaving every thing behind them.

If in this instance, instead of digging a well, the boring operations had been begun upon the surface of the ground, the water would have been raised at a much less expense, and with no danger to the workmen.

The water remained at the level of 12 metres above the bottom of the well. It comes from the chalk stratum which lies underneath the beds mentioned above, which differ from those in the neighbourhood of Calais only in the circumstance, that the marl and limestone with marine fossils do not exist in the latter locality. The clay beds are absolutely the same.

From all this we are led to believe, 1st, that the waters contained in the beds of clay lying underneath the formation of marine limestone, are the result of a gradual percolation through the earths above them; 2dly, that they have no communication with those which were forced up 30 feet above the heads of the workmen, since these latter rose and remained stationary at the height of 12 metres above the bottom of the well, while the former stood only at 5 metres above it; 3dly, that, if the water belonging to the

chalk formation was met with in the bed of siliceous and argillaceous sand, it was because the little crevices existing in the lower part of this bed, into which the water was forced by its upward pressure, were by this very pressure widened and opened until they reached the hard bed of clay with pyrites; that the waters thus found means of spreading throughout the argillaceous sand, and went no farther on account of the impermeability of the hard bed immediately above it.

At the Courtalin paper factory, after passing through a depth of 26 metres composed of sand, of clay more or less hard, and of marl, a stratum of chalk was met with containing little layers of flint. The water appeared the moment the chalk was entered, and rose to within 1.30 of the surface.

196. It may sometimes happen in boring, that we may meet with large bodies of water which come from the beds above the chalk; but they have generally a disagreeable taste and odor. Moreover, they are not subjected to a pressure sufficient to cause them to rise to the surface, because they have merely soaked through the recent formations, and do not come originally from elevated situations like those we find in the chalk stratum. These latter are generally perfectly

limpid and healthy, and care must be taken to give them a passage through the bad water from the upper beds without their communicating with it.

197. It is only in calcareous rocks, then, that we should seek for rising springs. We have seen, that these springs may be found wherever a stratum containing fissures sufficient to give passage to water, is enclosed between two others which are water-tight; and if the intermediate stratum comes to the surface in the more elevated spots so as to receive water from rain, and from streams, and afterwards descends without there being any vent for the waters, we have only to pierce through the upper bed and give to these a free passage, and they will rise to the surface of the ground and sometimes even above it. Such being the conditions, it may be affirmed that the chalk is the only stratum into which we should bore for water. In fact, it is well known that this formation is often contained between beds through which water cannot pass, that it crops out in the high lands, that it extends indefinitely at the bottom of the valleys, and that it is full of fissures which allow free passage to the water. The numerous caves which are found in limestone countries are an additional proof of the truth of this last remark.

It is useless to undertake boring in primitive formations containing granite, gneiss, porphyry serpentine, &c., as these rocks have but few fissures, and they are small. The water which soaks into them is soon stopped. Schistose rocks should be avoided also, because the ferruginous pyrites contained in them easily decompose, and communicate to the water the odor and taste of sulphuretted hydrogen.

198. It is important to observe that the auger may sometimes reach a body of water which will not, however, rise more than a few feet above the level at which it is found, although the fissures in which it stands may be constantly supplied from a very elevated source. For, if this water can find in a neighbouring valley, deeper than that in which the boring operations have been carried on, an issue large enough for its discharge, it will not rise at all; and in proportion as the issue is smaller, it will rise to a greater height.

But, even if the natural vent of the waters is very small, we may not find a spring if we make but a single attempt at boring; for we may hit upon a spot where the calcareous rock is compact in texture. A remarkable instance of this occurred at Béthune, not very far from Calais. A person undertook to bore for water, and after passing through 60 or 70 feet of recent formation,

and 30 feet of calcareous rock, met with a spring whose waters rose to the surface. His next-door neighbour endeavoured to do the same thing. He bored first through 70 feet of sand and grey clay containing a great quantity of pyrites, and 105 feet in a calcareous stratum, which was met with at the same depth as before. But though he had thus gone to a depth of 175 feet, he met with no water, and, being discouraged, abandoned the attempt. This undoubtedly shows that in the spot where the latter carried on his operations, the rock was homogeneous and compact. Had he gone to a greater depth, it is very probable that he would have met with some impermeable bed, from beneath which water would have risen to the surface. But this water evidently would have come from a different source from that of his neighbour.

Whenever, then, in boring, we meet with a homogeneous rock, it is necessary to continue our operations until we find a variation in its nature; for experience has shown that the water generally lies near the spot where the different formations meet. The cavities and fissures which generally exist there, facilitate its passage; and it is probably to this cause that we should attribute its greater abundance at this point. If we find, after entering a few feet into the chalk, that the water

will rise to the surface, we may confidently expect an increased quantity, if, in continuing to bore, we meet with layers of small rounded stones.

199. On the peninsula of Boston, there are wells which rise 75 or 80 feet above the level of the sea. Dr. Lathrop gave an account of them in 1800, in a paper published in the *Memoirs of the American Academy of Arts and Sciences.** The theory by which he accounts for the great rise in the waters is precisely the same as that given more at length by Garnier.

Dr. Lathrop gives the following history of a well dug near Boston Neck. "Where the ground was opened, the elevation is not more than one foot, or one foot and a half, above the sea at high water. The well was made very large. After digging about 22 feet in a body of clay, the workmen prepared for boring. At the depth of 108 or 110 feet the auger was impeded by a hard substance; this was no sooner broken through and the auger taken out, than the water was forced up with a loud noise, and rose to the top of the well. After the first effort of the long confined elastic air was expended, the water subsided about 6 feet from the surface and there remains at all seasons ebbing and flowing a little with the tides."

* Vol. III. pp. 57–67.

Dr. Lathrop observes, that the proprietors of this well were led to exercise great caution in carrying on the work, by an accident which had happened in their immediate neighbourhood. "A few years before, an attempt was made to dig a well a few rods to the east, near the sea. Having dug about 60 feet in a body of clay without finding water, preparation was made in the usual way for boring; and, after passing about 40 feet in the same body of clay, the auger was impeded by stone. A few strokes with a drill broke through the slate covering, and the water gushed out with such rapidity and force, that the workmen with difficulty were saved from death. The water rose to the top of the well and ran over for some time. The force was such as to bring up a large quantity of fine sand, by which the well was filled up many feet. The workmen left behind all their tools, which were buried in the sand, and all their labor was lost. The body of water which is constantly passing under the immense body of clay, which is found in all the low parts of the peninsula, and which forms the basin of the harbour, must have its source in the interior, and is pushed on with great force from ponds and lakes in the elevated parts of the country. Whenever vent is given to any of those subterranean currents, the water will rise, if it have opportunity, to the level of its source."

The reader cannot but remark how strikingly this last account coincides with that given by Héricart de Thury of the well at the Barrière de Fontainebleau.

APPENDIX.

In the foregoing Chapters we have had frequent occasion to change French measures into English; in so doing we have made use of the Table of constant multipliers given below, which has been calculated on the authority of the *Annuaire du Bureau des Longitudes* for 1832. If we multiply quantities of the denomination expressed in the first column of this Table by the corresponding numbers given in the second, the product will be the equivalent quantities expressed in units of the denomination given in the fourth column. Thus N metres $= N \times 39.37080$ English inches. The third column shows the logarithms of the numbers in the second.

1	2	3	4
Metres	39.37080	1.5951742	English inches
Metres	3.28090	0.5159930	English feet
Square metres	10.76431	1.0319860	Engl. square ft.
Cubic metres	35.31661	1.5479790	Engl. cubic ft.
French feet	1.06578	0.0276676	English feet
English inches	0.02540	0.4048258 — 2	Metres
English feet	0.30479	0.4840064 — 1	Metres
Engl. sq. feet	0.09290	0.9680140 — 2	Square metres
Engl. cub. feet	0.02832	0.4520210 — 2	Cubic metres
English feet	0.93828	0.9723324 — 1	French feet
Pouce d'eau	677.913	2.8311739	Engl. cubic feet in 24 hours.
Great Rom. ounce	1453.632	3.1624544	
Little Rom. ounce	726.816	2.8614244	
Quinaria	1977.730	3.2961670	
Double module	706.3322	2.8490090	

In all our calculations we have estimated *quantities* of water in cubic feet, adding also the *time* in which the given number of cubic feet was furnished. In France and in Italy a unit for measuring supplies of water has long been in use, which combines in itself the ideas both of *time* and of *quantity*. Such a unit is very desirable when water is to be distributed in a city, since the quantity to be furnished to the tenants consists always of a certain amount in a certain time. The *pouce d'eau*, which was long in use in France, is the quantity which flows in twenty-four hours by a circular orifice one French inch in diameter, with a head of one line above the upper edge of the orifice. The quantity thus defined was a very improper measure, since it would vary considerably under different circumstances, according to the thickness of the vessel in which the orifice was made, and as moreover it is extremely difficult to measure exactly so small a head. Hence different authors gave different values to it. Bossut found it to be 17.9388 cubic metres in twenty-four hours; Couplet, 18.2804; Mariotte, 19.7437. M. de Prony, whose experiments were superior in accuracy to those of his predecessors, fixed it at 19.1953, and this has of late years been almost universally adopted; so that a *pouce d'eau* may now be considered as a conventional term, meaning simply 19.1953 cubic metres in twenty-four hours.

M. de Prony says, in a paper printed in the *Memoirs of the Academy of Sciences* of Paris in 1817, that having been called upon to determine

some more convenient unit, better adapted to the decimal system of French measures, it seemed to him proper that this unit should bear a certain relation to the daily quantity required for each person. This quantity of course is very variable. The climate and the habits of the people have a great influence upon its determination; so that it is impossible to fix upon any value for it, which shall be universally applicable. M. de Prony, therefore, had particularly in view the climate of Paris, and the customs of its inhabitants, in deciding, as he did, upon ten cubic metres per day, as the quantity required for the domestic wants of 1000 persons. To this he gave the name of *module*, and twice this quantity, which is very nearly equal to the *pouce d'eau*, he called the double module. To determine the conditions under which exactly this quantity would flow from an aperture, was the important point; and for this purpose he entered upon a series of experiments, which were executed with remarkable accuracy. The ingenious apparatus which he invented, in order to maintain the surface of the water at a constant height above the orifice, is described in the paper above referred to. By means of it, he ascertained that a double module of water, or 20 cubic metres (706.33 cub. ft.) in 24 hours, will be discharged by a circular orifice having a radius of one centimetre (0.0328 ft.), the surface of the water being 5 centimetres (0.1640 ft.) above its centre, and the efflux being through an adjutage 17 millimetres (0.0558 ft.) in length.

M. de Prony also determined the value of t
modern Roman *ounces* as follows: the great Rom
ounce, 41.16 cubic metres in 24 hours; the lit
ounce, 20.58 cubic metres; the *quinaria* used
Ancient Rome (see Commentary of Frontinus on t
Roman Aqueducts), 56 cubic metres.

END.

CAMBRIDGE:
CHARLES FOLSOM, PRINTER TO THE UNIVERSITY.

Pl. I.

Fig. 5.

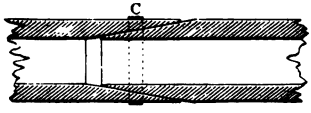


Fig 6

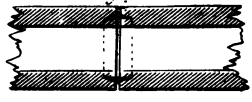


Fig. 7.

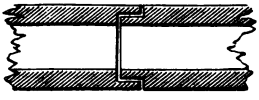


Fig. 8.

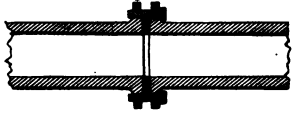


Fig. 9.

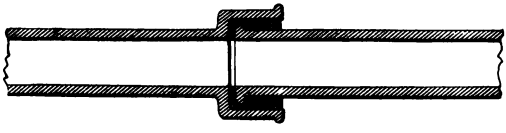


Fig. 2.

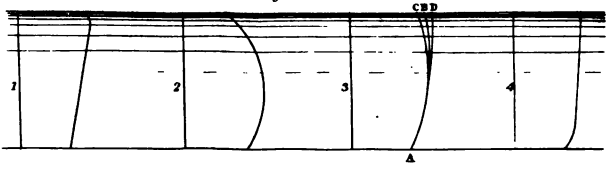


Fig. 3.

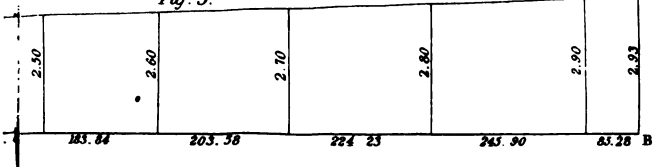
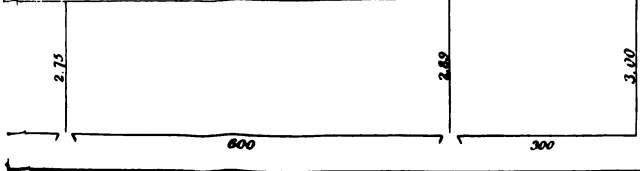
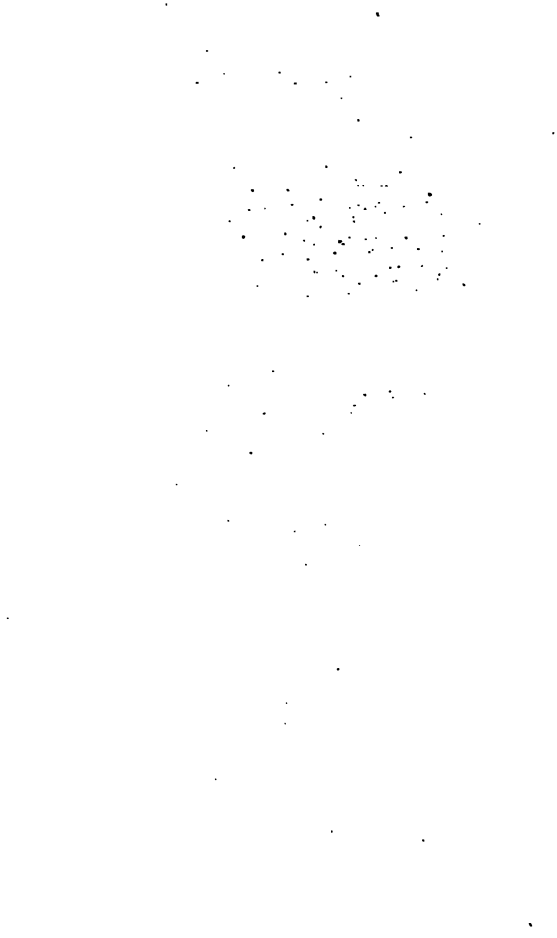
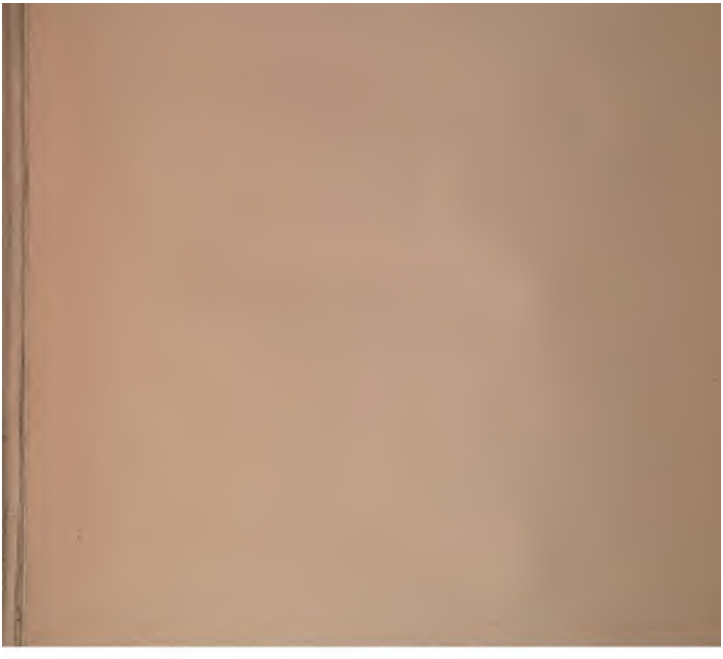


Fig. 4.









B'D. MAR 24 1914

